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PHASOR ANALYSIS OF TWO-STAGE GM-TYPE PULSE TUBE REFRIGERATOR

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ABSTRACT

A two-stage pulse tube refrigerator (PTR) is the simplest version of a multistage pulse tube refrigerator. A two-stage Gifford-McMahon (GM) has been used for nearly all of the cryopumps. The phasor analysis of the Gifford-McMahan (GM)-type pulse tube refrrigerator (PTR) have not explained detail in the earlier study. In the present work, a detailed mass flow, enthalpy & phasor analysis for a two-stage PTR, both for OPTR and DIPTR has been studied.

The phasor analysis helps to understand the importance of phase difference between mass flow rate at the cold end and the pressure pulse in the pulse tube. The refrigerating effect for different types of PTR strongly depends on the phase shift arrangement and also on the phase difference. Due to the complexities of the flow involved in the two-stage, the phasor diagram will prove to be very helpful to understand the importance of these parameters. It also introduces the concept and importance of phase lead of the double inlet flow entry in the OPTR & DIPTR.

Keywords: Pulse tube cryocooler; GM-type; Phassor analysis.

1. INTRODUCTION

Pulse tube refrigerators have demonstrated many advantages with respect to temperature stability, vibration, reliability and lifetime among small cryocoolers [1]. The pulse tube cryocooler is a very attractive for cryogenic cooling since the absence of moving parts in the cold part promises considerable system advantages with respect to reliability, lifetime, vibration and cost. After the introduction of the basic pulse tube refrigerator concept by Gifford and Longsworth in the mid 1960s, essential improvements of this cooler type have been achieved by two modifications: First, adding a reservoir (buffer volume) an orifice valve to the warm end of the pulse tube (orifice mode, orifice pulse tube cooler), and Second, adding a bypass valve connecting the warm end of the pulse tube to the main gas inlet (so-called double-inlet mode, double pulse tube cooler). These advances enable the pulse tube coolers to have a cooling performance comparable to Stirling and GM cryocoolers, indicating the great application potentials of the pulse tube cooler [2].These modifications serve to adjust the phase shift between pressure and mass flow oscillation.

Second, rare-earth based magnetic materials such Er_3Ni, $HoCu_2$ have been produced as small spheres, which exhibit increased specific heats at temperature 10K are used as regenerator materials in the second stage, to match the specific heat of helium gas at these temperature [3,4]. It is easier to achieve a cold end temperature below 20K for a two-stage high frequency pulse tube refrigerator than that of one-stage [1]. In the second stage regenerator is the key component . The phasor analysis is extended to a two stage orifice pulse tube refrigerator (OPTR) and to a double inlet pulse tube refrigerator (DIPTR) [5].

2. ANALYSIS FOR TWO STAGE OPTR

Figure1 shows the schematic diagram of a two stage pulse tube cryocooler (PTC) [4]. In the Gifford-McMahon (GM) type pulse tube refrigerator (PTR) a compressor produces high and low pressure continuously. In this configuration, the cold end of the first stage regenerator forms the warm end of the second stage regenerator. In the pulse tube both the stages are mounted separately at their warm ends (HX1 and HX2) on the base stainless steel flange.

The phase shifters for the first and second stages (double inlet valves, orifice valves and reservoir) are all located at room temperature.

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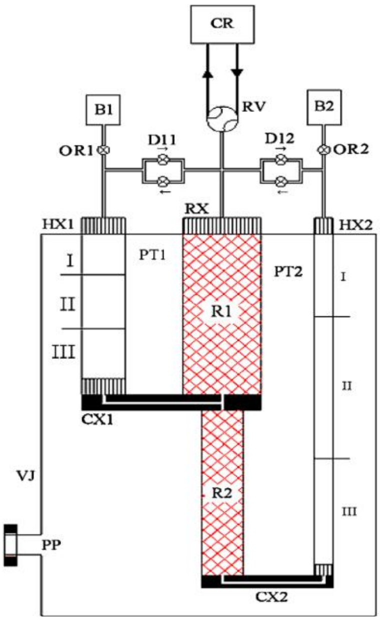

Figure1.

Schematic of the two stage pulse tube cryocooler (PTC), CR-Compressor; RV- Rotary valve; B1, B2- Buffer volume; OR1, OR2- Orifice valves; DI1, DI2- Double inlet valves; HX1, HX2- Heat exchanger at the Pulse tube hot end, CX1, CX2- Heat exchanger at the Pulse tube cold end, RX- Heat exchanger at the regenerator hot end (Aftercooler); R1, R2- Regenerator; PT1, PT2- Pulse tube; VJ- Vacuum jacket; PP- Evacuation port; [4]

Basic assumptions are as follows [6]:

1. The gas flow is laminar, no turbulence.
2. Constant wall temperature at the cold-and hot-end heat exchangers.
3. The regenerative material in the regenerator is incompressible, uniform porosity.
4. Boundary and variable permeability effects are neglected.

Under the assumptions, the governing equations are given by:

Continuity equation,

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0$$

Energy equation of the gas

$$(1-f)\rho\frac{\partial H}{\partial t} = -\rho u\frac{\partial H}{\partial x} + (1-f)\frac{\partial p}{\partial t} + \alpha F(T_r - T) + \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}).$$

Pressure drop of the gas,

$$\frac{\partial p}{\partial x} = -\eta z_r u.$$

Equation of state for real gas,

$$\rho = f(T, p).$$

The energy equation for the solid regenerative material in the regenerator is:

$$f\rho_r C_r \frac{\partial T_r}{\partial t} = \alpha F(T - T_r) + \frac{\partial}{\partial x}(k_r \frac{\partial T_r}{\partial x}).$$

Mass flow in the pulse tube

The mass flow through the valve is caused by the pressure difference between the pulse tube and reservoir. The time averaged pressure in the reservoir is not exactly the same as that in the pulse tube, because the mass flow oscillation through the valve boots the time average pressure in the reservoir. For the simplifying the mathematics involved in this analysis and understanding this phenomenon, the pressure in the pulse tube is assumed to be sinusoidal in shape and the pressure in the reservoir to be steady [7].

The pulse tube is divided into three parts; the gas in the cold flow to and from the regenerator, the gas of the middle part is reciprocated in the pulse tube, the gas in the hot part flow via a orifice valve and bypass valve. The regenerator is divided into two parts connected via a flow resistance. The assumptions are [8];

1. The pressure in the pulse tube and the cold part of the regenerator is equal and uniform.
2. The gas temperature in each part of the regenerator and the pulse tube is uniform and constant except in the middle parts of the pulse tube.
3. The behavior of the gas column in the middle part of the pulse tube is adiabatic.

The governing equations can be formulated are as follows using the mass conservation law,

$$\frac{d(\rho V_{pt})}{dt} = (\rho\dot{v})_c - (\rho\dot{v})_h \qquad (1)$$

Assuming that the pulse tube is at a mean temperature, density can be taken as $\rho = \frac{P}{RT_{mpt}}$

$$\frac{d(PV_{pt})}{RT_{mpt}dt} = \dot{m}_c - \dot{m}_h \qquad (2)$$

$$\dot{m}_h = \dot{m}_c - \frac{d(PV_{pt})}{RT_{mpt}dt} \qquad (3)$$

$$\frac{V_{pt}}{RT_{mpt}}\frac{dP}{dt} = \dot{m}_c - \dot{m}_h \qquad (4)$$

The mass flow rate is proportional to pressure difference,

$$\Delta P = P_1 \cos \omega t \qquad (5)$$

$$\frac{dP}{dt} = \omega P_1 \cos(\omega t + \frac{\pi}{2})$$

Mass flow rate at hot end is given as,

$$\dot{m}_h = \frac{T_h}{T_c} C_1 P_1 \cos \omega t \tag{6}$$

Putting the value of $\frac{dP}{dt}$ in equation (4),

$$\frac{\omega V_{pt}}{RT_{mpt}} P_1 \cos(\omega t + \frac{\pi}{2}) = \dot{m}_c - \dot{m}_h \tag{7}$$

Again putting the value of equation (6) in equation (7),

$$\frac{\omega V_{pt}}{RT_{mpt}} P_1 \cos(\omega t + \frac{\pi}{2}) = \dot{m}_c - \frac{T_h}{T_c} C_1 P_1 \cos \omega t$$

$$\dot{m}_c = \frac{T_h}{T_c} C_1 P_1 \cos \omega t + \frac{\omega V_{pt}}{RT_{mpt}} P_1 \cos(\omega t + \frac{\pi}{2}) \tag{8}$$

From equation (8), it is clear that mass flow rate at the cold end consists of two components. It was confirmed that the refrigerator displayed good temperature stability at the second stage cold end, though the mass flow rate from compressor might be slightly change during operation due to change in temperature of cooling water for the compressor. The first term in the right hand side is the contribution resulting from the mass flow rate at the hot end through the orifice which is in phase with the pressure. The second term represents the addition amount of mass entering the pulse tube due to pressurization.

Enthalpy flow

Applying the first law of thermodynamics to this system, the cold end heat exchanger of the pulse tube refrigerator.

$$\dot{Q} = (\dot{H}) - (\dot{H}_r)$$

For perfect regeneration, $(\dot{H}) = 0$

The enthalpy flows entering and leaving the system at the compressor side are equal, so there is no net enthalpy change due to this gas flow. Also there is no enthalpy flows entering and leaving this system at the regenerator side.

Hence the refrigerating effect obtained will be,

$$\dot{Q} = (\dot{H}) = \frac{C_P}{\tau} \int_0^{\tau} \dot{m}_c T dt \tag{9}$$

In the pulse tube, temperature variation are given as:

$$T = T_0 + T_1 \cos(\omega t) \tag{10}$$

Putting the value of *T* in equation (9),

$$(\dot{H}) = \frac{C_P}{\tau} \int_0^{\tau} (\dot{m}_c)(T_c + T_1 \cos \omega t) dt \tag{11}$$

Phasor analysis for two stage orifice pulse tube refrigerator (OPTR)

A phasor diagram for a pulse tube refrigerator (PTR) is a vectorial representation of mass flow rate, pressure, and temperature at different locations as a function of time. Figure. 2 shows the phasor diagram for second stage of the pulse tube. The phasor diagram for the second stage pulse tube of the two-stage OPTR where $\dot{m}_{h2}$ and $\dot{m}_{c2}$ represent the phasors for mass flow rate at the second stage hot and cold ends, respectively. The flow coming out of regenerator-1 gets divided into two streams; one stream goes to the first stage pulse tube and the second one goes through regenerators-2.Therefore, the mass flow rate through the cold end of the regenerator-1 is

$$\dot{m}_{Reg1c} = \dot{m}_{\mathrm{Re}g2h} + \dot{m}_{c1} \tag{12}$$

Mass flow at cold end is

$$\dot{m}_{c1} = \frac{T_h}{T_{c1}} C_1 P_1 \cos \omega t + \frac{\omega V_{pt1}}{RT_{mpt1}} P_1 \cos(\omega t + \frac{\pi}{2})$$

As shown in Figure (2), $\dot{m}_{\mathrm{Re}g2h}$ can be expressed as,

$$\dot{m}_{\mathrm{Re}g2h} = \frac{T_h}{T_{c2}} C_1 P_1 \cos \omega t + \frac{\omega V_{pt2}}{RT_{mpt2}} P_1 \cos(\omega t + \frac{\pi}{2})$$

$$+ \frac{\omega V_{\mathrm{Re}g2}}{RT_{m\mathrm{Re}g2}} P_1 \cos(\omega t + \frac{\pi}{2}) \tag{13}$$

In the figure 2 , A_1, A_2 & A_3 are expressed

$$A_1 = \frac{T_h}{T_{c2}} C_1 P_1 , \; A_2 = \frac{\omega V_{pt2}}{RT_{mpt2}} P_1 ,$$

$$A_3 = \frac{\omega V_{\mathrm{Re}g2}}{RT_{m\mathrm{Re}g2}} P_1$$

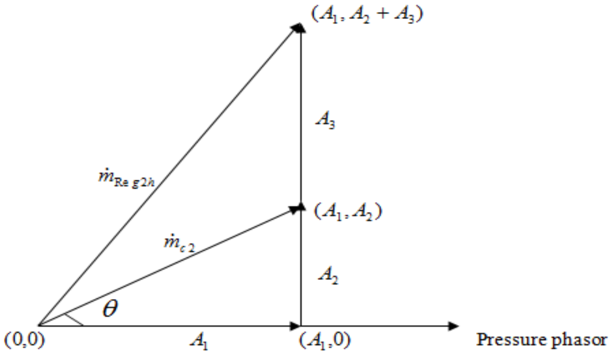

Figure 2: Phasor diagram for second stage OPTR

3. ANALYSIS FOR TWO STAGE DIPTR

The double inlet configuration yielded the highest cooling capacity for the single pulse tube cooler. A double inlet pulse tube refrigerator differs from orifice pulse tube refrigerator by a connection from the high pressure line to the hot end of the pulse tube. The double inlet pulse tube refrigerator, described in the subsequent section, provides the gas required to the charge the warm volumes directly from the after cooler and therefore reduces the amount of flow that the regenerator must process [9]. In double inlet pulse tube refrigerator, the wave form of flow velocity at the cold end of each stage can be obtained by linearly superimposing the flow velocity of the double inlet pulse tube on that of the orifice.

Mass flow in the double inlet pulse tube refrigerator (DIPTR)

Consider the whole pulse tube as a control volume and applying the law of conservation of mass [4],

$$\frac{d(\rho V_{pt})}{dt} = (\rho\dot{v})_c - (\rho\dot{v})_o + (\rho\dot{v})_{DI} \tag{14}$$

$$\frac{d(PV_{pt})}{RT_{mpt}dt} = \dot{m}_c - m_o + m_{DI} \tag{15}$$

$$\dot{m}_c - \frac{d(PV_{pt})}{RT_{mpt}dt} = \dot{m}_o - \dot{m}_{DI} \tag{16}$$

We know,

$\dot{m}_h = \dot{m}_c - \dfrac{d(PV_{pt})}{RT_{mpt}dt}$ from equation (3),

$$\dot{m}_h = \dot{m}_o - \dot{m}_{DI}$$

(17)

Where $\dot{m}_h$ is net mass flow rate, it is possible to get the optimum phase between pressure and the mass flow at the hot end of the pulse tube.

$$\frac{d(PV_{pt})}{RT_{mpt}dt} = \dot{m}_c - \dot{m}_h \tag{18}$$

From equation (6),

$$\dot{m}_h = \frac{T_h}{T_c} C_1 P_1 \cos\omega t \tag{19}$$

Thus, $\dot{m}_o$ and $\dot{m}_{DI}$ are directly proportional to the pressure difference across the valves. As per assumption, $\dot{m}_{DI}$ leads the pressure by phase angle *α*. Substituting $\dot{m}_h$ in equation (18);

$$\dot{m}_c = \frac{T_h}{T_c} C_1 P_1 \cos\omega t + \frac{\omega V_{pt}}{RT_{mpt}} P_1 \cos(\omega t + \frac{\pi}{2})$$
$$-\frac{T_h}{T_c} C_2 P_1 \cos(\omega t + \alpha) \tag{20}$$

Enthalpy flow

For all tested inserts the gross cooling power is reduce to some extend because of the regenerative effect of the insert material that reduces the enthalpy flow in the double inlet pulse tube refrigerator.

Substituting the mass flow rate expression from equation (20);

$$(\dot{H}) = \frac{C_p}{\tau}[\int_0^\tau \frac{T_h}{T_c} C_1 P_1 \cos\omega t (T_c + T_1 \cos\omega t) dt$$
$$-\int_0^\tau \frac{T_h}{T_c} C_2 P_1 \cos(\omega t + \alpha)(T_c + T_1 \cos\omega t) dt$$

The first term is the same that for the OPTR, while the second term represents the contribution due to the flow through the double inlet valve.

Phasor analysis for two stage double inlet pulse tube refrigerator (DIPTR)

The magnitude and phase of the mass flow rate at the hot end of the second stage regenerator can be obtained from Eq. (15). Mass should be conserved at the junction of the first stage and second stage, which means that the sum of the mass flow rate entering the hot end of the second regenerator and the mass flow rate entering the cold end of the first stage tube must be equal to the mass flow rate leaving the cold end of the first stage regenerator Figure (3) shows the phasor diagram of two stage DIPTR.

The mass flow rate through regenerator-1 is divided through pulse tube-1 and regenerator-2 which is given below,

$$\dot{m}_{\mathrm{Re}g1c} = \dot{m}_{\mathrm{Re}g2h} + \dot{m}_{c1}$$

The mass flow rate at the cold end of the pulse tube-1 $\dot{m}_{c1}$ can be redefined from Eq. (20) while $\dot{m}_{\mathrm{Re}g2h}$ is given as,

$$\dot{m}_{\mathrm{Re}g2h} = \frac{T_h}{T_{c2}} C_1 P_1 \cos\omega t + \frac{\omega V_{pt2}}{RT_{mpt2}} P_1 \cos(\omega t + \frac{\pi}{2})$$
$$-\frac{T_h}{T_{c2}} C_2 P_1 \cos(\omega t + \alpha) + \frac{\omega V_{\mathrm{Re}g2}}{RT_{m\mathrm{Re}g2}} P_1 (\omega t + \frac{\pi}{2})$$

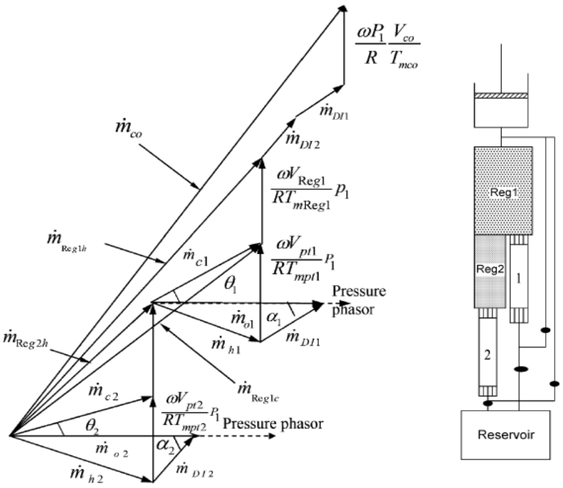

Figure 3.Phasor diagram corresponding to the two-stage DIPTR [7]

CONCLUSIONS

In the present work, detail mass flow rates are shown at the hot and cold ends of the pulse tube, and also the mass flow rate through the second stage orifice and double-inlet valve. Some predicted results of the phase difference of the gas temperature, the pressure, the mass flow rate and the enthalpy flow, in the second stage regenerator are presented in the paper. The phasor diagrams have been presented for two-stage orifice pulse tube refrigerator (OPTR) and double-inlet pulse tube refrigerator (DIPTR) from which the concept of phase difference and double inlet phase lead angle can be very well understood. It has been found that the refrigerating effect is directly proportional to the mass flow rate at the cold end and the amplitude of dynamic pressure.

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NOMENCLATURE

Symbol	
T	temperature (K)
p	pressure (Pa)
u	velocity (m/s)
ρ	density (kg/m^3)
η	viscosity (sPa)
k	thermal conductivity (W/K m)
τ	time period of pressure oscillation (s)
C_p	specific heat of helium gas (J/K kg)
t	time (s)
f	filling factor
H	enthalpy (J)
R	gas constant per unit mass (kJ/kg K)
α	heat transfer coefficient (W/K m^3)
F	heat exchange area per unit volume (m^2/m^3)
$\dot{m}_h$	mass flow rate at the hot end
$\dot{m}_c$	mass flow rate at the cold end
m_o	mass flow of the orifice valve
m_{DI}	mass flow of the double inlet valve
T_h, T_c	hot and cold temperatures (K)
V_{pt}	volume of the pulse tube (m^3)
V_{Reg1}, V_{Reg2}	volume of the first and second stage regenerator
$\dot{m}_{\mathrm{Re}g1c}$	mass flow rate at the cold end of the first stage regenerator
$\dot{m}_{\mathrm{Re}g2c}$	mass flow rate at the hot end of the first stage
$\dot{Q}$	refrigerating effect
$\dot{H}$	time-average enthalpy flow from pulse tube
$\dot{H}_r$	time average enthalpy flow from regenerator

Subscripts

PTR	pulse tube refrigerator
OPTR	orifice pulse tube refrigerator
DIPTR	double inlet pulse tube refrigerator
h	hot end
c	cold end
RG	regenerator

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