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Improved Methods for Superelevation Distribution: I. Single Curve

Udai Hassein[1], Said Easa[2], Kaarman Raahemifar[3]
[1]Ph.D Candidate, [2]Professor, Dept. of Civil Engineering, Ryerson University, Toronto, Canada
[3]Professor, Dept. of Electrical and Computer Engineering, Ryerson University, Toronto, Canada
uhassein@ryerson.ca, uhassein@gmail.com

Abstract- **The American Association of State Highway and Transportation Officials (AASHTO) presents 5 methods for distributing highway superelevation (e) and side friction factor (f). According to AASHTO, Method 5 (curvilinear) is better than Method 1 (linear) and deals with speed variations; however, its complex mathematical calculation makes it intractable and can affect design consistency. This paper proposes new methods to distribute superelevation and side friction using an equal-arc unsymmetrical parabolic curve (EAU Method) and a single-arc unsymmetrical curve (SAU Method).**

Reliability analysis is common used in field of transportation engineering. This paper developed a Reliability model based on the e distribution using the First-Order Second-Moments (FOSM) method. Reliability modeling descripts for the parameters' variability by utilizing the mean and standard deviation into a closed form estimation method. The distribution of e is then used to estimate the reliability index for the current standards. Using the reliability-based design method, transportation engineers can adjust horizontal design curves fulfilling a desired probability of failure for the reliability index β. Numerical examples are presented to show the advantages of the proposed methods over the AASHTO method.

Keywords-- **Highway design; Design consistency; Superelevation distribution; Reliability.**

I. INTRODUCTION

The horizontal alignment contains tangents, such as straight paths, having no curvature and circular curves that are horizontal, connecting the tangents with or without a transition spiral curve. The vertical alignment contains upgrade and downgrade tangents connected by parabolic curves. Other possible options for vertical alignments include unsymmetrical curves such as an equal-arc unsymmetrical parabolic curve (EAU) and a single-arc unsymmetrical curve (SAU). An EAU curve has two equal arcs. In addition, the curve contains uneven horizontal projections of the tangents (Easa, 1994). The SAU curve contains a single arc based on a cubic function.

The SAU curve lies above the flatter arc and under the sharper arc of the EAU vertical curve, thus, making it smoother (Easa, 2007).

The American Association of State Highway and Transportation Officials (AASHTO, 2004) presents 5 methods for distributing highway superelevation (e) and side friction (f) (Fig. 1). AASHTO applies Method 2 and 5 for low and high speeds, respectively, with distributing superelevation rates for speed variations along both urban and rural settings.

Method 5 focuses on technical qualities for both Methods 1 and 4. It also distributes intermediate superelevation rates among them, using complex unsymmetrical parabolic curves. This distribution aims to increase superelevation rates and the safety margin of compliant speed variation, which is unspecified in Method 1. It also satisfies the side friction factor of sharper curves to avoid irregular driving that can be ingrained within Method 4.

The goal of this study is to achieve alignment design consistency in a horizontal curve of a road that allows safe driving at a preferred speed (Krammes et al., 1995). Specifically, design inconsistency was assumed to show itself with the need to decelerate from the desired speed in order to safely negotiate certain alignment elements (Nicholson, 1998; Easa, 1999). Using the design speed of horizontal curves Easa (2003) developed an optimization model to improve highway design consistency based on safety margin.

This paper presents a modified AASHTO Method 5 using the equal-arc unsymmetrical parabolic curve and a single-arc unsymmetrical curve to form the basis of the EAU and SAU Methods, respectively. The proposed methods determine the best superelevation distribution for specified highway data by using the complete superelevation design region (of which AASHTO curves are a division). The following section presents AASHTO Method 5 of the superelevation distribution, which provides a useful background for what follows.

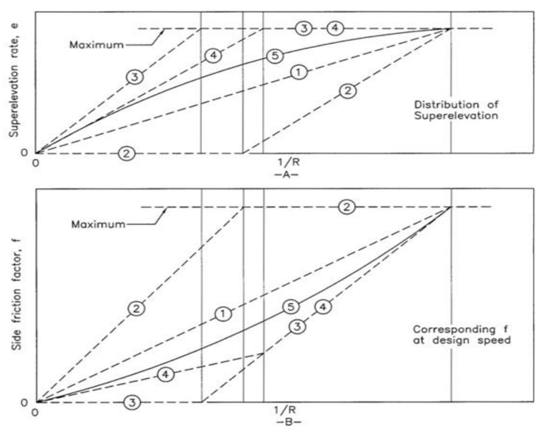

Fig.1. Representative Superelevation Distribution AASHTO Methods (AASHTO, 2004)

Reliability analysis is common used in field of transportation engineering. It has been used in intersection sight distance (Easa, 2000) and in pedestrian for traffic signals (Easa and Cheng, 2013). Reliability analysis utilizes highway design variables. In terms of safety margin, a performance function is utilized to calculate the reliability level. Reliability level of potential improvements and existing conditions is calculated in terms of probability of failure. A smaller probability of failure indicates a higher reliability level.

This paper developed a Reliability model based on the $\boldsymbol{e}$ distribution using the First-Order Second-Moments (FOSM) method. Reliability modeling descripts for the parameters' variability by utilizing the mean and standard deviation into a closed form estimation method. An application example of horizontal curves assessment that reliability analysis for superelevation distribution is provided by considering Abia's methodology (2010) and a sensitivity analysis for various random variables are presented.

Evaluating the AASHTO Methods

Side friction and superelevation regarding Above-Minimum Radius Curves, it is evaluated based on Nicholson (1998) procedure. Fig. 2 shows the different methods for distribution involving e and f over a set of available curve ranges.

The unsymmetrical (US) curve was first created for e based on the f distribution.The unsymmetrical curve deals with two parabolic arcs connected at the point of intersection ($1/R_{PI}$); whereas, R_{PI} is the radius analyzed utilizing Equation (1), where $e = e_{max}$ and $f = 0$.

Regardless of the difference between the average running speed and design speed, and by utilizing the basic principles of unsymmetrical vertical curves (Hickerson, 1964), side friction (f) can result in:

$$e + f = \frac{V^2}{gR} \qquad (1)$$

$$R_{PI} = \frac{V^2}{g\, e_{max}} \qquad (2)$$

For $1/R \leq 1/R_{PI}$

$$f_5 = f_{max} \frac{R_{min}\, R_{PI}}{2\, R^2} \qquad (3)$$

For $1/R > 1/R_{PI}$

$$f_5 = f_{max} \left[\frac{R_{min}}{R} \left(\frac{R_{PI} - R}{R_{PI} - R_{min}} \right) + \frac{R_{min} R_{PI}}{2\, R^2} \left(\frac{R - R_{min}}{R_{PI} - R_{min}} \right)^2 \right] \qquad (4)$$

Then, the superelevation, e_5, is set by:

$$e_5 = \frac{V^2}{g\, R} - f_5 \qquad (5)$$

For $e_{max} = f_{max}$, it is shown that f_5 and e_5 for Equation (1) – (5) is decreased to f_5 along with e_5 within the symmetrical curve.

II. Distribution Using Fixed Curves

EAU Method

The EAU method is a proposed mathematical procedure for superelevation distribution based on AASHTO's Method 5 and EAU curve equations. This section describes the derivation of the proposed method. Side friction factors and superelevation distribution rates are very important when designing appropriate horizontal highway alignments. Based on the laws of mechanics, superelevation rate (e) required for turning along a horizontal curve contributes to the following equation:

$$0.01e = \frac{V^2}{127\, R} - f \qquad (6)$$

Where, g = gravity constant (9.81 m/s^2), R = turning radius, v = vehicle speeds (km/h), and f = side friction factor. Practical design values exist regarding the upper limits of e, e_{max}, and f_{max}, taking into account many factors such as driving comfort, safety, pavement, traffic, and weather. According to the AASHTO, when traveling at a precise design speed the minimum turning radius (R_{min}) can be resolved such that both e_{max} and f_{max} are chosen.

This value functions as a maximum value for limiting both side friction factors and superelevation rates from what is considered realistic for either comfort or operation by drivers. Therefore, utilizing a radius larger than R_{min} permits both superelevation rates and side friction factors to have design values that are lower than their upper limit. The curve parameters ($R_{PI}, h_{PI}, g_1, g_2, L_1, L_2$) would be calculated according to AASHTO as follows:

The radius at the point of intersection (R_{PI}) can be obtained by:

$$R_{PI} = \frac{V_R^2}{1.27\ e_{max}} \quad (7)$$

Where, PI is the Point of Intersection, with legs (1) and (2) for the f distribution parabolic curve, and V_R = running speed (km/h). The PI counterbalance from $1/R$ axis, which can be obtained by:

$$h_{PI} = \left(\frac{(0.01\ e_{max})V_D^2}{V_R^2}\right) - 0.01\ e_{max} \quad (8)$$

The slope for leg (1), g_1, can be obtained by:

$$g_1 = h_{PI} * R_{PI} \quad (9)$$

The slope for leg (2), g_2 , can be obtained by:

$$g_2 = \frac{f_{max} - h_{PI}}{\frac{1}{R_{min}} - \frac{1}{R_{PI}}} \quad (10)$$

A is the algebraic difference between g_2 and g_1 , i.e., $A = g_2 - g_1$. The length of the first arc (L_1), the second arc (L_2), and the total length of the curve (L) can be obtained by Eq. 11 to 13.

$$L_1 = \frac{1}{R_{PI}} \quad (11)$$

$$L_2 = \frac{1}{R_{min}} - \frac{1}{R_{PI}} \quad (12)$$

$$L = L_1 + L_2 \quad (13)$$

Using the EAU vertical curve equation one can establish the following equation.

The curve parameter of the proposed EAU vertical curve equation is x and is given by

$$X = \frac{1}{R} \quad (14)$$

The parameter that describes the unsymmetrical curve (R_{EAU}) can be obtained by:

$$R_{EAU} = \frac{L_1}{L} \quad (15)$$

R_{EAU} is characterized as the ratio of the length of the shorter tangent (or a shorter arc when dealing with a traditional curve) to the length of the whole curve. The rate of change of grades for the first and second arcs (r_1 and r_2) can be determined using Eq. 16 and 17, respectively:

$$r_1 = \frac{A(3-4R_{EAU})}{L}, \quad L_1 < L_2 \quad (16)$$

$$r_2 = \frac{A(-1+4R_{EAU})}{L}, \quad L_1 < L_2 \quad (17)$$

Where, the value of x is measured from the vertical axis, and when $x \le L/2$, the first arc elevations for the EAU Curve, as $y_1 = f_1$ can be determined as:

$$f_1 = g_1 x + \frac{r_1 x^2}{2} \quad (18)$$

Note that y_{BVC}= 0 for f_1 where $f_1 = f$ distribution at any given point $x \le L/2$; and,

$$e_1 = \frac{V^2}{127\ R} - f_1 \quad (19)$$

Where $e_1 = e$ distribution at any given point $x \le L/2$.

When $x > L/2$, the second arc elevations for the EAU curve, as $y_2 = f_2$ can be obtained as follows:

$$f_2 = f_{max} - g_2 (L - x) + \frac{r_2 (L-x)^2}{2} \quad (20)$$

Note that $y_{EVC} = f_{max}$ for f_2 where f_2 is the f distribution at any given point $x > L/2$; and

$$e_2 = \frac{V^2}{127\ R} - f_2 \quad (21)$$

Where e_2 is the e distribution at any given point $x > L/2$.

SAU Method

The SAU Method is a proposed mathematical procedure for superelevation distribution based on AASHTO's Method 5 and the SAU Curve Equations. This section describes the derivation of the proposed method. When designing a suitable horizontal alignment for highways, superelevation rate distribution (or the equivalent turning radii/curvatures) and side friction factors are very important. The laws of mechanics and superelevation rates affect the drivers' ability to make a successful turn at a horizontal curve. The SAU Method of superelevation uses Eq. 7 to 14 to determine the curve parameters such as R_{PI}, h_{PI}, g_1 , g_2, L_1, L_2, L, A, and x .

Using the SAU vertical curve equation, one can establish the following:

The rate of change of the slope (r_{PVC}) can be obtained as:

$$r_{PVC} = \left(\frac{-2A}{L^2}\right)(L_1 - 2\,L_2) \qquad (22)$$

The constant t can be obtained by the following equation:

$$t = \left(\frac{6A}{L^3}\right)(L_1 - L_2) \qquad (23)$$

The value of x is measured from the vertical axis. When $x \leq 1/R_{PI}$, or $x > 1/R_{PI}$ the arc elevations for the SAU Curve, as $y = f$, can be determined as:

$$f = g_1 x + \frac{r_{PVC}}{2} x^2 + \frac{t}{6} x^3 \qquad (24)$$

Note that h_{PVC}= 0 for f, where h_{PVC} = the elevation of PVC; f is the distribution at any given point $x \leq 1/R_{PI}$, or $x > 1/R_{PI}$; and,

$$e = \frac{V^2}{127\,R} - f \qquad (25)$$

Where e is the distribution at any given point $x \leq 1/R_{PI}$, or $x > 1/R_{PI}$.

III. RELIABILITY METHOD

Reliability method accounts for parameter variation by using the mean and standard deviation in a closed form estimation method. Both the mean and standard deviation of the superelevation distribution used in the proposed model will be compared with Abia's model (2010) values for evaluation purposes. The FOSM method has been used in many applications to compute the reliability index and provide the probability of failure.

Reliability of superelevation for AASHTO Method 1

To avoid driving off the road and sliding involving vehicles that operate in designed speed, lateral forces need to be counterbalanced with the effect for the superelevation along with the frictional forces that are found on tires.

The derivation of design equations for friction factor and superelevation distribution would be considered according to by Abia's model (2010) as follows:

$$f = \frac{R_{min}}{R} f_{max};\ R_{min} = \frac{V^2}{g(e_{max}+f_{max})} \qquad (26)$$

$$e = \frac{V^2}{gR} - \frac{V^2}{gR}\left(\frac{f_{max}}{e_{max}+f_{max}}\right) = \frac{V^2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \qquad (27)$$

Side friction demands from drivers can be directly proportional with the lateral acceleration of a given speed $\boldsymbol{v}$, e, and with R.

Thus; a random quantity is normally distributed within the mean $\bar{f}$ and the variance σ_{fs}^2. The speed can also have a random quantity along with it being normally distributed within the mean v and with the variance σ_v^2. Given that these two quantities can be random variables, the probability density functions is generated and the function is used for the reliability analysis of e. Reverting back to simplified curve equations that are transposed of e, both the 1st and the 2nd partial derivatives for the e-functions will be:

e-function:

$$e = \frac{V^2}{gR} - \frac{V^2}{gR}\left(\frac{f_{max}}{e_{max}+f_{max}}\right) = \frac{V^2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \qquad (28)$$

Partial derivatives:

$$\frac{\partial e}{\partial v} = \frac{2\,v}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \qquad (29)$$

$$\frac{\partial^2 e}{\partial v^2} = \frac{2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \qquad (30)$$

$$\frac{\partial^2 e}{\partial v \partial f} = 0 \qquad (31)$$

$$\frac{\partial e}{\partial f} = 0 \qquad (32)$$

$$\frac{\partial^2 e}{\partial f^2} = 0 \qquad (33)$$

The following task needs to be applied to the above formulation towards the superelevation equation. This can be done by the following: Based on the equation (28), and assuming both v and f are suitable probability distribution functions, expected values with variance for the necessary superelevation rate, e can be obtained. Initially, applying Taylor's theory towards e expression that uses the 2nd order approximation, the formula above may be shown as:

$$e \approx \frac{\bar{v}^2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) + \frac{2\bar{v}}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right)(v-\bar{v}) + \frac{\partial e}{\partial f}(f-\bar{f}) + \frac{\partial^2 e}{\partial v^2}\frac{(v-\bar{v})^2}{2!} + \frac{\partial^2 e}{\partial v \partial f} cov(v,f) + \frac{\partial^2 e}{\partial f^2}\frac{(f-\bar{f})^2}{2!} = \frac{\bar{v}^2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) + \frac{2\bar{v}}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right)(v-\bar{v}) + \frac{2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) + \frac{(v-\bar{v})^2}{2!} \qquad (34)$$

After generalization,

$$e \approx \frac{\bar{v}^2 + 2\bar{v}(v-\bar{v})}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \qquad (35)$$

The expected value for $E(e)$ can be obtained with placing an expected value operator (E) within the right side of the term to the 2nd order approximation , which produces:

$$E(e) \approx \frac{\overline{v}^2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right)+\frac{2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) E\,\frac{(v-\overline{v})^2}{2!} \quad (36)$$

Replacing the speed variance, $(v-\overline{v})^2$ with a symbol of σ_v^2, the expected value can be:

$$E(e) = \frac{\overline{v}^2+\sigma_v^2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \quad (37)$$

The e varianceis obtained with the following only using the 1st order approximation once the information connecting the third moment and the forth moment for the underlying variables may not be available.

$$\sigma_e^2 \approx \frac{4\overline{v}^2\sigma_v^2}{(gR)^2}\left(\frac{e_{max}}{e_{max}+f_{max}}\right)^2 \quad (38)$$

The Reliability Index Safety may be rewritten with the expected value ratio for e towards the standard deviation for e. Therefore:

$$\beta_e = \frac{\mu_e}{\sigma_e} = \frac{\overline{v}^2+\sigma_v^2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \div \frac{2\,\overline{v}\sigma_v}{(gR)}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \quad (39)$$

With the Failure Probability

$$P_f = P\,(e < e_{req}) \quad (40)$$

or

$$P_f = 1 - \emptyset\left[\frac{\mu_e}{\sigma_e}\right] = 1 - \emptyset(\beta_e) \quad (41)$$

Where; $\emptyset$ becomes the CDF for the standard normal variate along with $\emptyset^{-1}(1-P_f)$ being the value for the standard normal variate having the probability level $(1-P_f)$.

From reliability analysis, the confidence level (1- α) and e_{req} can be calculated by:

$$e_{req} = E(e) + Z_\alpha\,\sigma_\alpha \quad (42)$$

$$e_{req} = \frac{\overline{v}^2+\sigma_v^2+2\,z_\propto\overline{v}\sigma_2}{127R}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \quad (43)$$

As for $e_{req} \leq e_{max}$, minimum required curve radius, R_{req} can be calculated by Abia's model:

$$R_{req} = R_{min}\left(\frac{\overline{v}^2+\sigma_v^2+2\,z_\propto\overline{v}\sigma_2}{v_d^2}\right) \quad (44)$$

When $R_{min} = R_{req}$, $e_{req} = e_{max}$. $\overline{v}$ = average running speed and σ_V = standard division of average running speed can be calculated by Abia's model:

$$\overline{v} = 0.9749\,V_{85} - 3.6758\,;\ R^2 = 0.993 \quad (45)$$

$$\sigma_{\overline{v}} = 1.3821 + 0.7333\,(V_{85}-\overline{v})\,;\ R^2 = 0.712 \quad (46)$$

Improved model for Reliability of Superelevation

Based on Abia (2010), the reliability analysis application towards superelevation design was shown, which demonstrated that the reliability approach was straightforward for applying and producing superelevation rates, which are logically comparable with Method 5 and with the NCHRP 439 distribution model. Modern concepts within highway designs are highlighted through incorporating safety factors or incorporating reliability indexes within the design. Lastly, it was shown that ignoring the speed variations may lead to a considerable underestimation for the necessary superelevation rates that will place higher risks for drivers as they are cornering curves.

Using the Taylor approximation, Abia (2010) developed the "*e*" function, and he utilized it only up to the second term of approximation to calculate the superelevation. The authors, in this paper, considered the third term of approximation and showed that better and more reliable results are possible. In this study, the third term was used to calculate the superelevation to use it as a required superelevation for the reliability analysis, as shown in Equation (47).

$$e \approx \frac{\overline{v}^2+2\overline{v}(v-\overline{v})}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right)+\frac{2}{gR}\left(\frac{e_{max}}{e_{max}+f_{max}}\right) \quad (47)$$

$$e \approx (\bar{v}^2 + 2\bar{v}\,(v-\bar{v}) + (v-\bar{v})^2$$

$$\approx (\bar{v} + (v-\bar{v})^2)\left(\frac{e_{max}}{e_{max}+f_{max}}\right)\left(\frac{1}{gR}\right) \quad (48)$$

The expected value for $E(e)$ may be obtained with placing an expected value operator (E) within the right side of the term to the 2nd order approximation, as shown in Equation 36. Replacing the speed variance, $(v-\overline{v})^2$ with a symbol of σ_v^2, the expected value can be illustrated in Equation (37).

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The e variance is:

1) Obtained with the following method only using the 1st order approximation once the information connecting the third moment and, the fourth moment for the underlying variables may not be available, shown in Equation 38.
2) The Reliability Index Safety shown in Equation 39 highlights the expected value ratio for $\boldsymbol{e}$ towards the standard deviation for $\boldsymbol{e}$. The Probability of Failure is shown in Equation 41.

The summary of reliability method shows that Abia (2010) used side friction factor and the speed as random variables and the selected radius (R) and the reliability confidence level (z) are equal to 95% and 98.2%, respectively. Furthermore, also selected were e_{max} and R design, which calculated the e required based on the reliability analysis. The e required then calculates the R required, which can be solved by mathematical software such as MATLAB. This application for these equations having superelevation design can be demonstrated within the next section.

IV. Applications

The purpose of this section is to present numerical examples that demonstrate improvements in the distribution of the side friction factor and superelevation rates for a single curve using EAU and SAU Methods. Examples 1, 2 and, 3 show the distribution of superelevation rate obtained by AASHTO Method 5, EAU Method and SAU Method, respectively.

Numerical Example 1 (AASHTO's Method 5)

Using the AASHTO Method 5 for calculating e with a design speed of 80 km/h and $e_{\max}$ of 8 percent is V_R = 70 km/h, and f_{max} = 0.14 (the highest allotted side friction factor). The results are as follows: R_{min} = 228.945, R_{PI} = 482.038, h_{PI} = 0.02449, S_1 = 11.805, S_2 = 50.368, L_1 = 0.00207, L_2 = 0.00229, and L = 0.00437. The middle ordinate (MO) is 0.02090. The e distribution value of a radius can be determined by obtaining the $(0.01\,e+f)_D$ value and deducting either one of the f_1 or f_2 values (Fig. 2).

Therefore, the e distribution value of R = R_{PI} would result in $(0.01\,e+f)_D$ = $V_D^2/127\,R$ = 0.1045 minus an f_1 = 0.045, which makes e_1= 0.059 or 5.9 %. The e value is calculated for R = 482.038 m at 80 km/h design speed.

Table 1 shows the parameters for developing the superelevation distribution based on AASHTO's Method 5 using emax of 8% with speed ranging from 40 km/h to 130 km/h.

Table 1.
Parameters for Developing the Superelevation Distribution used for examples 1, 2 and 3

VD km/h	VR km/h	e_{max} %	f_{max}	R_{min} (m)	R_{PI} (m)	h_{PI}	L_1	L_2	g_1	g_2	M.O.
20	20	8	0.18	12.11	39.35	0.000	0.025	0.057	0.000	3.148	0.028
30	30	8	0.17	28.33	88.54	0.000	0.011	0.024	0.000	7.083	0.027
40	40	8	0.17	50.37	157.40	0.000	0.006	0.014	0.000	12.592	0.027
50	47	8	0.16	81.98	217.31	0.011	0.005	0.008	2.290	19.675	0.025
60	55	8	0.15	123.18	297.58	0.015	0.003	0.005	4.525	28.332	0.023
70	63	8	0.14	175.29	390.45	0.019	0.003	0.003	7.327	38.563	0.022
80	70	8	0.14	228.95	482.04	0.024	0.002	0.002	11.805	50.368	0.021
90	77	8	0.13	303.56	583.27	0.029	0.002	0.002	17.086	63.747	0.019
100	85	8	0.12	393.50	710.76	0.031	0.001	0.001	21.839	78.700	0.018
110	91	8	0.11	501.19	814.64	0.037	0.001	0.001	30.056	95.227	0.015
120	98	8	0.09	666.64	944.79	0.040	0.001	0.000	37.745	113.328	0.012
130	102	8	0.08	831.27	1023.49	0.050	0.001	0.000	51.124	133.003	0.008

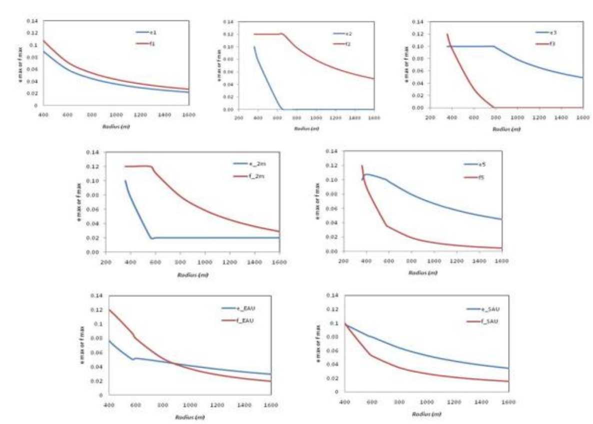

Fig. 2. Comparison of Superelevation Distribution Methods

Numerical Example 2 (EAU Method)

Using the EAU Method for calculating e with a design speed of 80 km/h and e_{max} of 8 percent is, V_R = 70 km/h, f_{max} = 0.14 (the highest allotted side friction factor). The results are as follows: R_{min} = 228.945, R_{PI} = 482.038, h_{PI} = 0.02449, g_1 = 11.805, g_2 = 50.368, A = 38.563, L_1 = 0.00207, L_2 = 0.00229, L = 0.00437, R_{EAU} = 0.47495, r_1 = 9713.344, r_2 = 7944.303.

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The e distribution value of a radius can be determined by obtaining e_1 and e_2 values and deducting either one of the f_1 or f_2 values, respectively (Fig. 2).

Therefore, the e distribution value is determined when $R = R_{PI}$ resulting in f_1 = 0.045 and e_1 = 0.059 or 5.91%. Table 2 shows the calculations of the distribution of the superelevation based on the EAU Method using e_{max} of 8% with speed ranging from 40 km/h to 130 km/h.

Table 2.
The EAU Method to Design Superelevation Rates for Different Radii (e_{max} = 8%)

Design Speed (km/h)	40	50	60	70	80	90	100	110	120	130
Radius (m)	e	e	e	e	e	e	e	E	e	e
7000	0.2	0.2	0.4	0.4	0.5	0.7	0.8	0.9	1.1	1.2
5000	0.2	0.3	0.6	0.6	0.8	0.9	1.1	1.3	2.3	1.7
3000	0.4	0.6	0.9	1.0	1.2	1.5	1.8	2.1	3.7	2.8
2500	0.5	0.7	1.1	1.2	1.5	1.8	2.1	2.5	4.5	3.4
2000	0.6	0.8	1.4	1.5	1.8	2.2	2.6	3.0	5.6	4.3
1500	0.8	1.1	1.8	1.9	2.4	2.8	3.4	4.0	7.4	5.8
1400	0.9	1.2	1.9	2.1	2.5	3.0	3.6	4.2	7.9	6.3
1300	0.9	1.3	2.1	2.2	2.7	3.2	3.9	4.5	8.5	6.8
1200	1.0	1.4	2.2	2.4	2.9	3.4	4.1	4.8		
1000	1.2	1.6	2.6	2.8	3.4	4.0	4.8	5.6		
900	1.3	1.8	2.9	3.1	3.7	4.4	5.2	6.2		
800	1.5	2.0	3.2	3.4	4.1	4.8	5.7	6.8		
700	1.7	2.3	3.6	3.8	4.5	5.3	6.3	7.5		
600	1.9	2.6	4.2	4.3	5.1	6.0	7.0			
500	2.3	3.1	4.9	4.9	5.8	6.7	7.9			
400	2.8	3.7	5.8	5.7	6.6	7.6				
300	3.6	4.6	7.2	6.7	7.5	8.3				
250	4.2	5.3	8.2	7.2	7.7					
200	4.9	6.0								
175	5.4	6.5								
150	6.0	6.9								
140	6.2	7.0								
130	6.5	7.1								
120	6.7	7.1								
110	6.9									
100	7.1									
90	7.3									
80										
70										
60										
50										
40										
30										
20										

Numerical Example 3 (SAU Method)

Using the SAU Method for calculating e with a design speed of 80 km/h and e_{max} of 8 percent is V_R = 70 km/h, f_{max} = 0.14 (highest allotted side friction factor).

The results are as follows: R_{min} = 228.945, R_{PI} = 482.038, h_{PI} = 0.02449, g_1 = 11.805, g_2 = 50.368, A = 38.563, L_1 = 0.00207, L_2 = 0.00229, r_{PVC} = 10155.604, t = -607520.746. The e distribution value of a radius can be determined by obtaining the e value and deducting the f value, respectively (Fig.2).

Therefore, the e distribution value is determined when $R = R_{PI}$, resulting in f = 0.045 and e = 0.059 or 5.9 %. Table 3 shows all the calculations for distribution of superelevation based on the SAU Method using e_{max} of 8% with speed ranging from 40 km/h to 130 km/h. Fig. 3. Shown the design superelevation rates for a maximum superelevation rate of 8% based on SAU Method.

Table 3.
The SAU Method to Design Superelevation Rates for Different Radii (e_{max} = 8%)

Design Speed (km/h)	40	50	60	70	80	90	100	110	120	130
Radius (m)	e	e	e	e	e	e	e	e	e	e
7000	0.2	0.2	0.3	0.4	0.5	0.7	0.8	0.9	1.1	1.2
5000	0.2	0.3	0.5	0.6	0.7	0.9	1.1	1.3	1.5	1.7
3000	0.4	0.6	0.8	1.0	1.2	1.5	1.8	2.1	2.5	2.9
2500	0.5	0.7	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5
2000	0.6	0.8	1.1	1.5	1.8	2.2	2.6	3.1	3.7	4.4
1500	0.8	1.1	1.5	1.9	2.3	2.8	3.4	4.0	4.9	5.7
1400	0.9	1.2	1.6	2.0	2.5	3.0	3.6	4.2	5.2	6.1
1300	0.9	1.3	1.7	2.2	2.7	3.2	3.9	4.5	5.5	6.4
1200	1.0	1.4	1.8	2.4	2.9	3.4	4.1	4.8	5.9	6.8
1000	1.2	1.6	2.2	2.8	3.4	4.0	4.8	5.6	6.7	7.6
900	1.3	1.8	2.4	3.0	3.7	4.4	5.2	6.1	7.2	7.9
800	1.5	2.0	2.6	3.4	4.0	4.8	5.7	6.6	7.6	7.9
700	1.7	2.2	3.0	3.8	4.5	5.3	6.3	7.1	7.9	
600	1.9	2.6	3.4	4.2	5.0	5.9	6.9	7.7		
500	2.3	3.0	3.9	4.9	5.7	6.7	7.5	7.9		
400	2.8	3.6	4.6	5.7	6.6	7.5	7.9			
300	3.5	4.5	5.7	6.8	7.5	7.9				
250	4.0	5.1	6.3	7.4	7.9					
200	4.8	5.9	7.1	7.8						
175	5.2	6.4	7.5	7.9						
150	5.8	6.9	7.8							
140	6.0	7.1	7.9							
130	6.3	7.3	7.9							
120	6.6	7.5	7.9							
110	6.8	7.7								
100	7.1	7.8								
90	7.4	7.9								
80	7.6	7.9								
70	7.8	7.8								
60	7.9	7.8								
50	7.9	8.1								
40	8.1									
30										
20										

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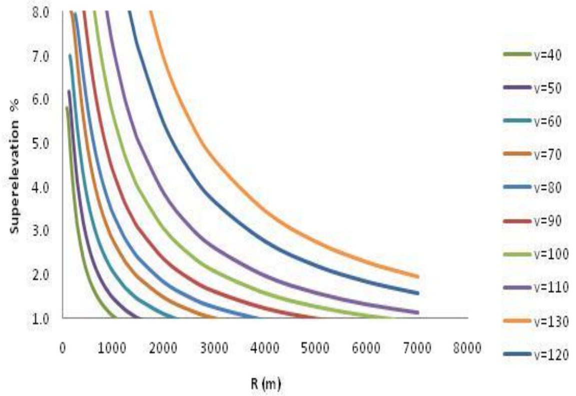

Fig. 3. Design Superelevation Rates for a Maximum Superelevation Rate of 8%

Numerical Example 4 (Reliability Method)

The proposed model was developed to obtain e distributions. MATLAB software was used to perform random sampling with probability density functions that have been chosen to have different parameters based on the adopted formulation. The data were used from Abia (2010) as an example. The required input parameters are: the design speeds (V_D) in (km/h); the running speeds (V_R) in (km/h); R = horizontal curve radius in meter; e_{max} of 8 percent; f_{max} = side friction factor; Z1 = 1.645 for 95% confidence level; and Z2 = 2.326 for 99% confidence level; calculate the required superelevation rates for 95% and for 99% confidence levels, respectively. The results for e required determined by Abia's model, and the proposed model were calculated as shown in Table 4.

Using the FOSM reliability method for calculating e with a design speed of 70 km/h and e_{max} of 8 percent is V_R = 64.6 km/h, f_{max} = 0.15 (highest allotted side friction factor). Therefore, the e distribution value for 95% confidence level is 0.02927 or 2.93 %, and the e distribution value for 99% confidence level is 0.03184 or 3.32 %. The results are as follows: R_{min} = 168 m, R_{req} for 95% confidence level = 182.73 m, R_{req} for 99% confidence level = 198.89 m. The reliability index (β_e) is 6.06, the probability of failure (P_f) is 0.001, and the reliability (R) is 99.9%.

Table 4 shows all the calculations for distribution of superelevation based on the FOSM Method using curve radius (R) 500 m, e_{max} of 8% , and speed ranging from 40 km/h to 130 km/h.

Comparison of Methods

The differences in the AASHTO, EAU, and SAU Methods can be seen in Fig. 2. The results show that:

1) For Methods 1, 2, and modified 2, f is larger than e for all radii.
2) For AASHTO Methods 3 and 5 and the EAU and SAU Methods, f is larger than e for small radii, while e is larger than f for large radii.
3) AASHTO Methods 1 and 5, along with the EAU and SAU Methods, provide a smooth change for e and f when the radius increases.
4) AASHTO Method 1 maintains the proportions for the centripetal force offered by the e and f constant.

The utilization of a minimum superelevation (similar to Method 2 Modified) involves introducing superelevation wherever it may presently not be offered. Although this approach will increase the construction costs, it could lead to fewer situations where vehicles lose control and slide along the inside curve crossing the centerline, and then, going from a difficult to favourable curvature. Therefore, decision-making is needed when a curve radius is very large in order to utilize the normal crossfall and tangent. Fig. 3 shows an example of developed superelevation for e = 8% using SAU Method.

V. Discussion

Regarding a single curve, the percentage of difference in e and f between AASHTO Method 5 and the EAU Method is less than the difference between AASHTO Method 5 and the SAU Method for the horizontal curve radius.

Based on Fig.4, for a horizontal curve with a large radius there will be a large decrease for a superelevation design. Conversely, the increase in side friction will be small. Furthermore, for a horizontal curve with a small radius, there will be a small reduction for a superelevation design; however, the increase in side friction will be very small. For the purpose of evaluation, these methods can be used to improve design consistency.

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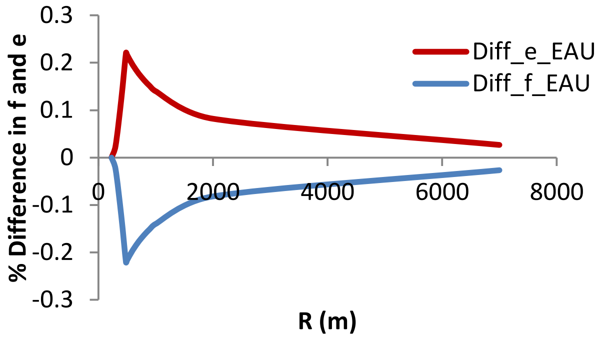
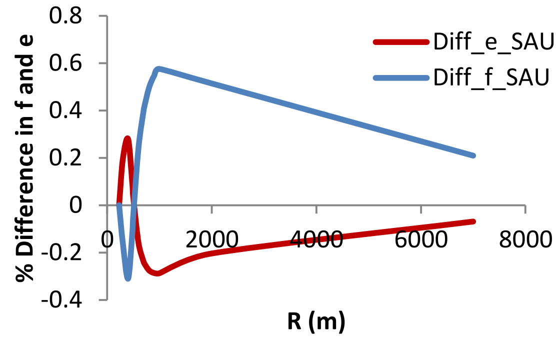


(a) The Difference Between AASHTO's Method 5 and the EAU Method



(b) The Difference Between AASHTO's Method 5 and the SAU Method

Fig.4. The Difference Between AASHTO's Method 5 and the Proposed Method (e_{max} = 8%)

Table 4.
Specifications of Design Speed, Average Running Speed and Superelevation

V	$\bar{V}$	$\sigma_{\bar{V}}$	R	e_{max}	f_{max}	Abia model		Proposed model		$R_{\min}$	R_{req}	R_{req}	B_e
(V_D)	(V_R)	(σ_{V_R})	(m)			e_{req} (95%)	e_{req} (99%)	e_{req} (95%)	e_{req} (99%)	(m)	(95%)	(99%)	
40	35.3	4.8	500	0.08	0.23	0.00744	0.00838	0.00748	0.00839	41	46.48	52.37	3.74
50	45.1	5.0	500	0.08	0.19	0.01305	0.01448	0.01309	0.01449	73	81.58	90.52	4.56
60	54.8	5.2	500	0.08	0.17	0.01999	0.02194	0.02003	0.02195	113	124.93	137.11	5.34
70	64.6	5.4	500	0.08	0.15	0.02924	0.03182	0.02927	0.03183	168	182.73	198.89	6.06
80	74.3	5.6	500	0.08	0.14	0.03957	0.04279	0.03961	0.04280	229	247.34	267.45	6.73
90	84.1	5.7	500	0.08	0.13	0.05211	0.05605	0.05214	0.05606	304	325.67	350.29	7.36
100	93.8	5.9	500	0.08	0.12	0.06717	0.07193	0.06720	0.07194	394	419.79	449.56	7.96
110	103.6	6.1	500	0.08	0.11	0.08515	0.09086	0.08519	0.09087	501	532.19	567.86	8.52
120	113.3	6.3	500	0.08	0.09	0.11281	0.12000	0.11285	0.12002	667	705.08	750.02	9.04
130	123.1	6.5	500	0.08	0.08	0.14020	0.14874	0.14024	0.14876	832	876.26	929.63	9.54

For reliability analysis, the differences between the results by Abia's model (2010) and the proposed model showed that:

1) The differences in the percentage of the required superelevation (e_{req}) are higher by using 95% level of confidence than the differences used by using 99% level of confidence.
2) Using different radiuses of curve will not effect on the differences in required superelevation therefore it can be concluded that the speed will have more effect on the differences in required superelevation than the radius of the curve.
3) The R required (R_{req}) and the superelevation (e) is larger within the proposed model compared to Abia's model (2010), as shown in Table 4. The difference in e % between Abia's model and the proposed model is shown in Figure 5.

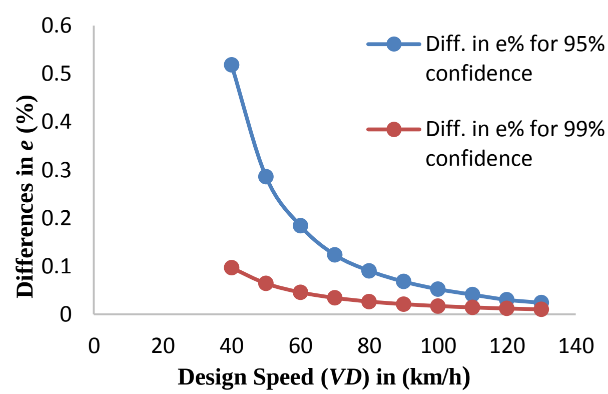


Fig. 5. Differences in % of superelevation for each Speed

VI. CONCLUSION

The proposed methods of superelevation design are not complicated to apply and will provide superelevation rates that are relatively comparable with AASHTO Method 5. As discussed earlier, the use of Method 5 signifies a mathematical analysis with little consideration for speed variation, along with an extensive process needed to attain the superelevation distribution.

The proposed methods, both EAU and SAU, have advantages over the AASHTO methods. The proposed methods bring into consideration only curvilinear alignments. These results certify a superelevation design that will assist most drivers with a low level of certainty. The information provided will be helpful for both the evaluator and designer, especially when discussing design sufficiency. Integrating the safety index with the turning radius design, along with the superelevation distribution, promises a thorough proposed method analysis built into the design methodology.

For the highway geometric design within the horizontal alignment element, the focus is on vehicle motion dynamics within the superelevated horizontal curve. In the vertical alignment element, the benefit is to discuss the derivatives of the EAU and SAU curve equations. The cross-section element considered with superelevation emphasizes the contribution toward vehicle stability.

For reliability analysis, AASHTO is stated that using Method 1 accounting for side friction factor distribution is reasonable; however, uniform speed assumption may put drivers at risk once they corner a curve. Using the reliability approach deals with the speed variation, therefore, it eliminates the expectation for constant speed, which is a major setback of Method 1. Additionally, a user needs to be warned of the necessary minimum turning radius due to reliability constraints are more conservative than R_{min} that is defined within Method 5.

When comparing Method 5, most of the reliability-based superelevations within the 95% confidence level are 1% - 2% lower within the design speed and with the curve radius. New concepts within highway designs are highlighted through incorporating safety factors or incorporating reliability indexes within the design.

Future work needs to consider the safety margin to improve the design consistency of highways using a superelevation distribution.

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