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BER of Chaotic-Conventional Spread Spectrum System

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***Abstract*— In this paper, the performance of chaotic-conventional digital communication system is studied at different spreading ratio between the conventional spreader and the chaotic spreader. This situation may be proposed when the enhancement in existing conventional spreading system is required by using the chaotic spreader while the conventional system is still in used. The analytical expressions of bit error rate (BER) for both the chaotic and the conventional spread spectrum receivers are derived. Finally an example is proposed to examine the behavior of the proposed system under the variation of the different system parameters.**

***Index Terms*— Bit Error Rate (BER), chaos, chaos communication, Spread spectrum communication.**

I. Introduction

Recently many communication systems are based on code division multiple access (CDMA) technology. The CDMA communication systems are spread-spectrum systems and make use of most of the modern communication and information-theoretic techniques that have so far been discovered by so many scientists and engineers. The spread spectrum technique was invented to adapt the military application but now it has been adapted for commercial applications.

CDMA spread spectrum systems can be divided into two types: frequency hopped and direct sequence. CDMA using frequency hopping involves a user transmitting over multiple frequencies consecutively in time in a pseudorandom manner. The Direct-sequence systems work by modulating the user's information signal with PN sequence known to the receiver and transmitter. This sequence is generated at a much higher rate than the user signal, literally "spreading" the user's signal bandwidth [1,2]. All commercial cellular CDMA systems use direct sequence spreading as opposed to frequency hopping.

The CDMA spread-spectrum systems have many advantages of that are high resistance for interference, high antijamming properties, unlimited users capacity and provide high security communication. These properties make CDMA systems attractive in modern communication applications.

The spreading code that used in CDMA is a subject for many researches to enhance the previous properties of the system. The trend is that "the autocorrelation of spreading code should tend to delta-function, the cross-correlation of different codes should tend to zero, and the code should be generated by simple and stable electronic circuits".

The chaotic signals provide new class of signals which can be used in spreading communication systems. The chaotic signal has significant properties [3,4,5]. It is much like noise signal, broadband signal, predictable in the short time and it is impossible to predict in long time run, and also it tends to ideal correlation property.

Many researches are carrying out to enhance the performance of CDMA systems [11]. The idea of combining chaotic signal with CDMA signal was ignited by Francis C. M. Lau (*et al*) [6], they proposed to use chaotic signals beside the conventional CDMA technique. This novel transmission technique is useful for using the advantages of chaotic signals and in the same time the conventional CDMA receivers are still in service.

The Coexistence should be designed under two constrains. First the performance of current CDMA receivers should not be degraded by applying the proposed transmission signal. Second the chaotic receivers should not be degraded also by conventional CDMA signal. This paper extends the results of [6] through studying the performance of conventional spread spectrum receiver with coexistence of chaotic signal and proved that constrain of no degradation satisfied.

The proposed system was studied in [6] under the condition of the spreading factor, defined as the number of samples transmitted in one binary symbol duration, of chaotic spreader equals the PN spreader. This paper will expand the result for different spreading factor between the chaotic and PN spreaders.

This paper is organized as follow. The conventional-chaotic system has been described in Section II. The performance analysis of the conventional and chaotic receivers has been explained in Section III. The example for proposed chaotic-conventional spread spectrum system is introduced in Section IV.

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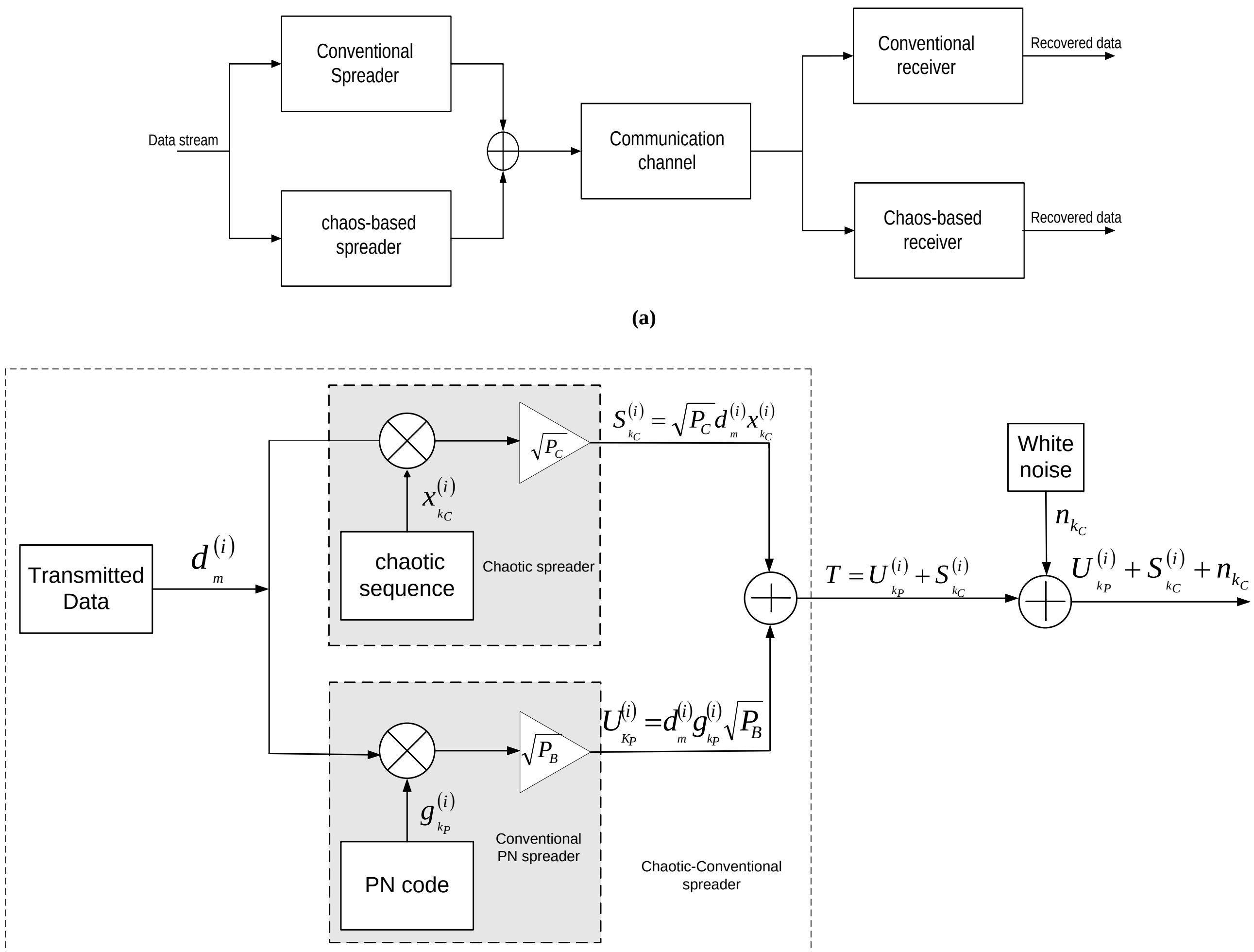
Conclusions are given in Section V. Appendix is attached in the end of this paper to derive some useful formulas that has been used through the paper.

II. System Description

Firstly suppose the system as in Fig. 1(a). The transmitter includes two spreaders; one for chaotic signal and the other for conventional PN code.



(a)

(b)

Fig. 1. The Chaotic-conventional spread spectrum system (a) block diagram for proposed transmitter and receiver (b) detail diagram showing the signal flow in the proposed transmitter system

The transmitted data is multiplied by each spreading signal individually and then the two spreaded signals are summed and transmitted. In the reception side, there is a possibility to use two different receivers, one uses conventional PN de-spreader and the second one uses chaotic de-spreader.

Now suppose we have L transmitters and the i<u>th</u> transmitter transmits a binary symbol $d_m^{(i)}$ during the m<u>th</u> bit duration with duration time equals T_S, where the symbol $d_m^{(i)}$ is either "+1" or "-1" each with equal probability and $i = 1,2,\ldots L$, as shown in Fig. 1(b).

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The chaotic sequence and PN code are used to spread the transmitted symbol $d_m^{(i)}$. All L transmitters have the same chaotic generator but each one has a unique initial condition to generate a unique chaotic sequence [3,7,12]. As a result there are L extremely different chaotic sequences each one is assigned to one transmitter. The i$^{\underline{th}}$ chaotic generator produces the chaotic samples $\{x_{k_C}^{(i)}\}$ for $k_C = 1,2,..$ which are used to spread the binary sequence $\{d_m^{(i)}\}$ during the m$^{\underline{th}}$ bit duration.

Whereas The PN signal generator produces PN sequence with samples $\{g_{k_P}^{(i)}\}$ for $k_P = 1,2,\ldots$

Let B_c is the spreading factor of the chaotic spreader, which is simply the number of chaotic samples transmitted in one binary symbol duration. Assuming the spreading factor of chaotic spreader is multiple of that of PN spreader, so

$$B_c = NB \quad (1)$$

Where B is the spreading factor of PN spreader and $N = 1,2,3,\ldots$

Acccording to Fig.1(b) The output signal from chaotic spreader at transmitter i$^{\underline{th}}$ is

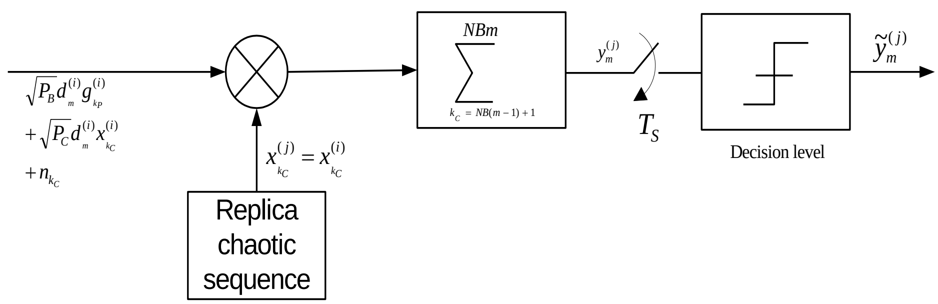

Fig. 2. The coherent chaotic receiver

$$S_{k_C}^{(i)} = d_m^{(i)} x_{k_C}^{(i)} \sqrt{P_C} \quad (2)$$

Where $\sqrt{P_C}$ is the power added to control the total power of chaotic spreaded signal. Similarly, the output signal from PN spreader is

$$U_{k_P}^{(i)} = d_m^{(i)} g_{k_P}^{(i)} \sqrt{P_B} \quad (3)$$

Where $\sqrt{P_B}$ is the power added to control the total power of PN spreaded signal, and the relation between k_P and k_C is as

$$k_P = \begin{cases} k_C & \text{if } N=1 \\ \left\lfloor \dfrac{k_C + (N-1)}{N} \right\rfloor & \text{if } N=2 \\ \left\lfloor \dfrac{k_C + (N-2)}{N} \right\rfloor & \text{if } N=3 \\ \vdots & \vdots \end{cases} \quad (4)$$

Where $\lfloor x \rfloor$ denotes the greatest integer smaller than or equal to *x*, so the transmitted signal is the summation of (2) and (3)

$$T = U_{k_P}^{(i)} + S_{k_C}^{(i)} \quad (5)$$

to increase the reality, the white noise is added to the transmitted signal; in this case the received signal at the receiver front end can be represented by:

$$\begin{aligned} R &= U_{k_P}^{(i)} + S_{k_C}^{(i)} + n_{k_C} \\ &= d_m^{(i)} g_{k_P}^{(i)} \sqrt{P_B} + d_m^{(i)} x_{k_C}^{(i)} \sqrt{P_C} + n_{k_C} \end{aligned} \quad (6)$$

Where n_{k_C} is the Gaussian noise sample of zero mean and variance $N_o/2$.

III. SYSTEM PERFORMANCE

A. Coherent chaotic receiver

The coherent chaotic receiver for j user is shown in Fig. 2, where the received signal is correlated with replica chao- -tic sequence, the correlation output for $m^{\underline{th}}$ bit is

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$$y_m^{(j)} = \sqrt{P_C}\, d_m^{(i)} \sum_{k_C=NB(m-1)+1}^{NBm} x_{k_C}^{(j)}.x_{k_C}^{(i)} + \sqrt{P_B}\, d_m^{(i)} \sum_{k_C=NB(m-1)+1}^{NBm} g_{kp}^{(i)} x_{k_C}^{(j)} + \sum_{k_C=NB(m-1)+1}^{NBm} n_{k_C} x_{k_C}^{(j)} \quad (7)$$

By assuming the chaotic sequences is known at receiver side we can write

$$x_{k_C}^{(i)} = x_{k_C}^{(j)} \quad (8)$$

So (7) can be rewritten as

$$y_m^{(j)} = \sqrt{P_C}\, d_m^{(i)} \sum_{k_C=NB(m-1)+1}^{NBm} (x_{k_C}^{(i)})^2 + \sqrt{P_B}\, d_m^{(i)} \sum_{k_C=NB(m-1)+1}^{NBm} g_{kp}^{(i)} x_{k_C}^{(i)} + \sum_{k_C=NB(m-1)+1}^{NBm} n_{k_C} x_{k_C}^{(i)} \quad (9)$$

The expected value (mean value) of $y_m^{(j)}$ is

$$E[y_m^{(j)}] = \sqrt{P_C}\, d_m^{(i)} \sum_{k_C=NB(m-1)+1}^{NBm} E[(x_{k_C}^{(i)})^2] + \sqrt{P_B}\, d_m^{(i)} \sum_{k_C=NB(m-1)+1}^{NBm} E[g_{kp}^{(i)}]E[x_{k_C}^{(i)}] + \sum_{k_C=NB(m-1)+1}^{NBm} E[n_{k_C}]E[x_{k_C}^{(i)}] \quad (10)$$

The average power of Gaussian noise, PN code equal to zero and also the chaotic sequence is selected with mean value equals to zero to avoid dc transmitted power when there is no information to transmit , so at this case

$$E[n_{k_C}] = E[g_{kp}^{(i)}] = E[x_{k_C}^{(i)}] = 0 \quad (11)$$

Thus (10) can be rewritten as

$$E[y_m^{(i)}] = \sqrt{P_C}\, NB.E[(x_{k_C}^{(i)})^2].d_m^{(i)} = \sqrt{P_C}\, E_b.d_m^{(i)} \quad (12)$$

Where $E_b = NB.P_S$ denotes the average bit energy of the chaotic system and P_S is the average power of chaotic signal $E[(x_{k_C}^{(i)})^2]$ [8]. Equation (12) explains that the transmitted data $d_m^{(i)}$ can be recovered at the receiver end whereas a positive scalar value ($\sqrt{P_C}\, E_b$) is multiplied by $d_m^{(i)}$.

Herein, we will study the BER for chaotic receiver. Suppose a binary symbol "+1" is transmitted during m symbol duration, so $d_m^{(i)} = +1$, and

$$y_m^{(j)} \big| \left(d_m^{(i)} = +1\right) = A + B + C \quad (13)$$

Where

$$A = \sqrt{P_C} \sum_{k_C=NB(m-1)+1}^{NBm} (x_{k_C}^{(i)})^2 \quad (14)$$

$$B = \sqrt{P_B} \sum_{k_C=NB(m-1)+1}^{NBm} g_{kp}^{(i)} x_{k_C}^{(i)} \quad (15)$$

$$C = \sum_{k_C=NB(m-1)+1}^{NBm} n_{k_C} x_{k_C}^{(i)} \quad (16)$$

The mean of equation (12) is

$$E\left[y_m^{(j)} \big| \left(d_m^{(i)} = +1\right)\right] = E[A] + E[B] + E[C] = E\left[\sqrt{P_C} \sum_{k_C=NB(m-1)+1}^{NBm} (x_{k_C}^{(i)})^2\right] + E\left[\sqrt{P_B} \sum_{k_C=NB(m-1)+1}^{NBm} g_{kp}^{(i)} x_{k_C}^{(i)}\right] + E\left[\sum_{k_C=NB(m-1)+1}^{NBm} n_{k_C} x_{k_C}^{(i)}\right]$$

$$= NBP_s \sqrt{P_C} \quad (17)$$

And the variance of $y_m^{(j)} \big| \left(d_m^{(i)} = +1\right)$ is [6,8]

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$$\operatorname{var}\left[y_m^{(j)} \middle| \left(d_m^{(i)} = +1\right)\right] = \operatorname{var}[A] + \operatorname{var}[B] + \operatorname{var}[C] + 2\operatorname{cov}[AB] + 2\operatorname{cov}[BC] + 2\operatorname{cov}[AC] \quad (18)$$

Where $\operatorname{cov}[XY]$ is the covariance between X and Y defined as [9]

$$\operatorname{cov}[XY] = E[XY] - E[X]E[Y] \quad (19)$$

It is easy to prove that $\operatorname{cov}[AC]$, $\operatorname{cov}[BC]$, and $\operatorname{cov}[AB]$ are zero as shown in the Appendix, so (18) can be reduced to

$$\operatorname{var}\left[y_m^{(j)} \middle| \left(d_m^{(i)} = +1\right)\right] = \operatorname{var}[A] + \operatorname{var}[B] + \operatorname{var}[C] \quad (20)$$

also from the Appendix

$$\operatorname{var}[A] = P_C NBE[(x_{k_C}^{(i)})^4] - \sqrt{P_C}\, NBP_S{}^2 = NB\left(P_C\beta - P_S^2\sqrt{P_C}\right) \quad (21)$$

Where

$$\beta = E[(x_{k_C}^{(i)})^4] \quad (22)$$

Also we can deduce that

$$\operatorname{var}[B] = P_B NBP_S \quad (23)$$

$$\operatorname{var}[C] = NB\frac{N_o}{2}P_S \quad (24)$$

For the m^th symbol, an error occurs when $y_m^{(j)} \le 0 \middle| \left(d_m^{(i)} = +1\right)$. Since $y_m^{(j)} \middle| \left(d_m^{(i)} = +1\right)$ is the sum of a large number of random variables, we may assume that it follows a normal distribution. Thus the error probability is given by [10]

$$\Pr ob\left(y_m^{(j)} \le 0 \middle| \left(d_m^{(i)} = +1\right)\right) = \frac{1}{2} erfc\left(\frac{E\left[y_m^{(j)} \middle| \left(d_m^{(i)} = +1\right)\right]}{\sqrt{2\operatorname{var}\left[y_m^{(j)} \middle| \left(d_m^{(i)} = +1\right)\right]}} \right) \quad (25)$$

So

$$\Pr ob\left(y_m^{(j)} \le 0 \middle| \left(d_m^{(i)} = +1\right)\right) = \frac{1}{2} erfc\left(\frac{NBP_S\sqrt{P_C}}{\sqrt{2NB\left(P_C\beta - P_S^2\sqrt{P_C}\right) + 2NBP_B P_S + NBN_o P_S}} \right) \quad (26)$$

Where $erfc(.)$ is error complementary function defined as [9]

$$erfc(x) \equiv \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-\lambda^2} d\lambda \quad (27)$$

As similar if the "-1" symbol is sent, the probability of error equals to

$$\Pr ob\left(y_m^{(j)} \ge 0 \middle| \left(d_m^{(i)} = -1\right)\right) = \frac{1}{2} erfc\left(\frac{-E\left[y_m^{(j)} \middle| \left(d_m^{(i)} = -1\right)\right]}{\sqrt{2\operatorname{var}\left[y_m^{(j)} \middle| \left(d_m^{(i)} = -1\right)\right]}} \right) \quad (28)$$

Where

$$E\left[y_m^{(j)} \middle| \left(d_m^{(i)} = -1\right)\right] = -NBP_s\sqrt{P_C} \quad (29)$$

And

$$\operatorname{var}\left[y_m^{(j)} \middle| \left(d_m^{(i)} = -1\right)\right] = \operatorname{var}[A] + \operatorname{var}[B] + \operatorname{var}[C] \quad (30)$$

And hence

$$\Pr ob\left(y_m^{(j)} \le 0 \middle| \left(d_m^{(i)} = -1\right)\right) = \frac{1}{2} erfc\left(\frac{NBP_S\sqrt{P_C}}{\sqrt{2NB\left(P_C\beta - P_S^2\sqrt{P_C}\right) + 2NBP_B P_S + NBN_o P_S}} \right) \quad (31)$$

The overall error probability of the m^th transmitted symbol in chaotic system in combined chaotic-conventional system is

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$$BER^{m}_{chaotic} = \Pr ob\left(d^{(i)}_{m} = +1\right) \times \Pr ob\left(y^{(j)}_{m} \le 0 \middle| \left(d^{(i)}_{m} = +1\right)\right) + \Pr ob\left(d^{(i)}_{m} = -1\right) \times \Pr ob\left(y^{(j)}_{m} \ge 0 \middle| \left(d^{(i)}_{m} = -1\right)\right) \quad (32)$$

Because there is an equal probability between "0" and "1" in transmitted signal, (32) can be rewritten as

$$BER^{m}_{chaotic} = \frac{1}{2}\left[\Pr ob\left(y^{(j)}_{m} \le 0 \middle| \left(d^{(i)}_{m} = +1\right)\right) + \Pr ob\left(y^{(j)}_{m} \ge 0 \middle| \left(d^{(i)}_{m} = -1\right)\right)\right] \quad (33)$$

$$= \frac{1}{2} erfc\left(\frac{NBP_S\sqrt{P_C}}{\sqrt{2NB\left(P_C\beta - P_S^2\sqrt{P_C}\right) + 2NBP_BP_S + NBN_oP_S}} \right) \quad (34)$$

The previous equation is independent on the transmitted m[th] symbol, so the error probability for the m[th] symbol is the same as the BER for the system.

$$BER_{chaotic} = BER^{m}_{chaotic} = \frac{1}{2} erfc\left(\frac{NBP_S\sqrt{P_C}}{\sqrt{2NB\left(P_C\beta - P_S^2\sqrt{P_C}\right) + 2NBP_BP_S + NBN_oP_S}} \right) \quad (35)$$

The last expression gives the analytical BER for the noisy coherent chaotic system under the interference of a conventional spread spectrum system. Note that for fixed P_C, the BER can be improved by:

- Selection of chaotic sequence with low β
- Increasing the spreading factor NB.
- Increasing the ratio P_S / P_B

B. Coherent conventional receiver

In conventional receiver, the received signal is multiplied by synchronized replica PN code as shown in Fig. 3, at this case

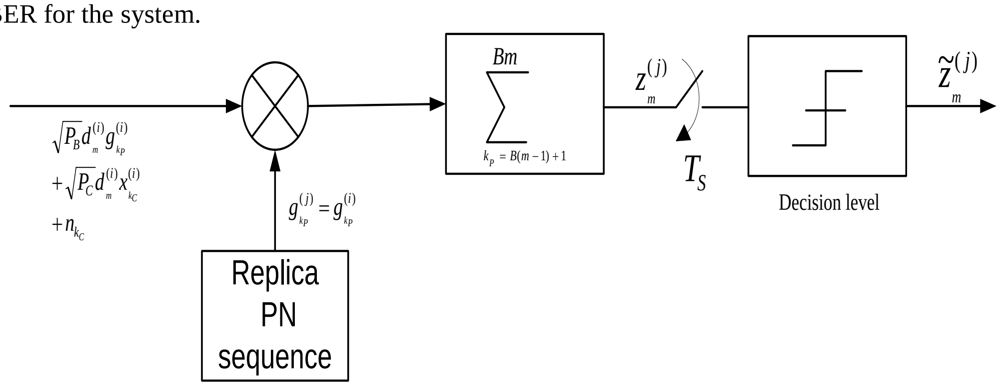

Fig. 3. The block diagram for conventional receiver

$$z^{(j)}_{m} = \sqrt{P_C} d^{(i)}_{m} \sum_{k_P = B(m-1)+1}^{Bm} x^{(i)}_{k_C} g^{(i)}_{k_P} + \sqrt{P_B} d^{(i)}_{m} \sum_{k_P = B(m-1)+1}^{Bm} (g^{(i)}_{k_P})^2 + \sum_{k_P = B(m-1)+1}^{Bm} n_{k_C} g^{(i)}_{k_P} \quad (36)$$

And suppose a binary symbol "+1" is transmitted during m symbol duration, so $d^{(i)}_{m} = 1$, so

$$z^{(j)}_{m} = A' + B' + C' \quad (37)$$

Where

$$A' = \sqrt{P_C} \sum_{k_P = B(m-1)+1}^{Bm} x^{(i)}_{k_C} g^{(i)}_{k_P} \quad (38)$$

$$B' = \sqrt{P_B} \sum_{k_P = B(m-1)+1}^{Bm} (g^{(i)}_{k_P})^2 \quad (39)$$

$$C' = \sum_{k_P = B(m-1)+1}^{Bm} n_{k_C} g^{(i)}_{k_P} \quad (40)$$

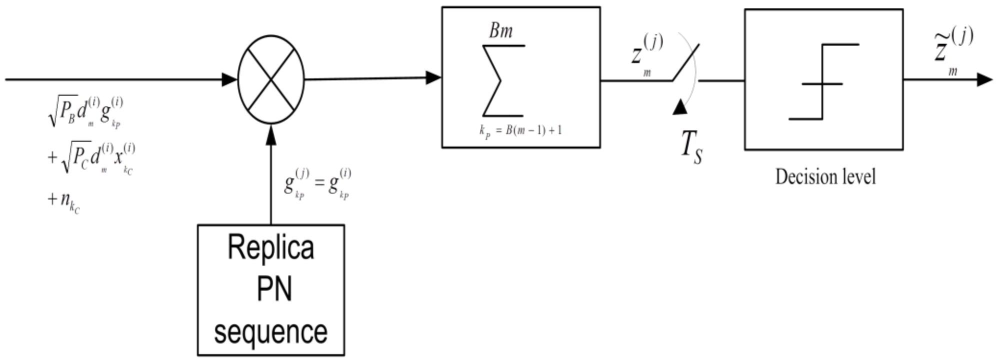

Fig. 3. The block diagram for conventional receiver

So the average of (36) is

$$E\left[z_m^{(j)}\middle|\left(d_m^{(i)}=+1\right)\right]=E[A']+E[B']+E[C']$$

$$=E\left[\sqrt{P_C}\sum_{k_P=B(m-1)+1}^{Bm}x_{k_C}^{(i)}g_{k_P}^{(i)}\right]+E\left[\sqrt{P_B}\sum_{k_P=B(m-1)+1}^{Bm}(g_{kp}^{(i)})^2\right]$$

$$+E\left[\sum_{k_P=B(m-1)+1}^{Bm}n_{k_C}g_{kp}^{(i)}\right]=B\sqrt{P_B} \qquad (41)$$

And the variance of $z_m^{(j)}\left|\left(d_m^{(i)}=+1\right)\right.$ is

$$\mathrm{var}\left[z_m^{(j)}\middle|\left(d_m^{(i)}=+1\right)\right]=\mathrm{var}[A']+\mathrm{var}[B']+\mathrm{var}[C']$$
$$+2\,\mathrm{cov}[A'B']+2\,\mathrm{cov}[B'C']$$
$$+2\,\mathrm{cov}[A'C'] \qquad (42)$$

As similar for formulas of coherent chaotic receiver which is derived in the Appendix, we can deduce that $\mathrm{cov}[A'C']$, $\mathrm{cov}[B'C']$, and $\mathrm{cov}[A'B']$ are zero, so (42) can be reduced to

$$\mathrm{var}\left[z_m^{(j)}\middle|\left(d_m^{(i)}=+1\right)\right]=\mathrm{var}[A']+\mathrm{var}[B']+\mathrm{var}[C'] \quad (43)$$

and

$$\mathrm{var}[A']=BP_CP_SE[(g_{k_p}^{(i)})^2]=BP_CP_S \qquad (44)$$

$$\mathrm{var}[B']=BP_B\,\mathrm{var}[(g_{k_p}^{(i)})^2]=0 \qquad (45)$$

$$\mathrm{var}[C']=B\frac{N_o}{2}E[(g_{k_P}^{(i)})^2]=B\frac{N_o}{2} \qquad (46)$$

For the m[th] symbol, an error occurs when $z_m^{(j)}\leq 0\left|\left(d_m^{(i)}=+1\right)\right.$. Since $z_m^{(j)}\left|\left(d_m^{(i)}=+1\right)\right.$ is the sum of a large number of random variables, we assume that it follows a normal distribution. The error probability is thus given by

$$\Pr ob\left(z_m^{(j)}\leq 0\middle|\left(d_m^{(i)}=+1\right)\right)=$$
$$\frac{1}{2}erfc\left(\frac{E\left[z_m^{(j)}\middle|\left(d_m^{(i)}=+1\right)\right]}{\sqrt{2\,\mathrm{var}\left[z_m^{(j)}\middle|\left(d_m^{(i)}=+1\right)\right]}}\right) \qquad (47)$$

So

$$\Pr ob\left(z_m^{(j)}\leq 0\middle|\left(d_m^{(i)}=+1\right)\right)=\frac{1}{2}erfc\left(\frac{B\sqrt{P_B}}{\sqrt{2BP_CP_S+BN_o}}\right) \quad (48)$$

As similar if the "-1" symbol is sent, probability of error equals to

$$\Pr ob\left(z_m^{(j)}\geq 0\middle|\left(d_m^{(i)}=-1\right)\right)=$$
$$\frac{1}{2}erfc\left(\frac{-E\left[z_m^{(j)}\middle|\left(d_m^{(i)}=-1\right)\right]}{\sqrt{2\,\mathrm{var}\left[z_m^{(j)}\middle|\left(d_m^{(i)}=-1\right)\right]}}\right) \qquad (49)$$

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Where

$$E\left[z_m^{(j)}\left|\left(d_m^{(i)}=-1\right)\right.\right]=-B\sqrt{P_B} \quad (50)$$

$$\operatorname{var}\left[z_m^{(j)}\left|\left(d_m^{(i)}=-1\right)\right.\right]=\operatorname{var}[A']+\operatorname{var}[B']+\operatorname{var}[C'] \quad (51)$$

And hence

$$\Pr ob\left(z_m^{(j)}\le 0\left|\left(d_m^{(i)}=-1\right)\right.\right)= \frac{1}{2}erfc\left(\frac{B\sqrt{P_B}}{\sqrt{2BP_CP_S+BN_o}}\right) \quad (52)$$

The overall error probability of the m[th] transmitted symbol in conventional system in combined chaotic-conventional system is

$$BER^m_{conventional}=\Pr ob\left(d_m^{(i)}=+1\right)\times \Pr ob\left(z_m^{(j)}\le 0\left|\left(d_m^{(i)}=+1\right)\right.\right) + \Pr ob\left(d_m^{(i)}=-1\right)\times \Pr ob\left(z_m^{(j)}\ge 0\left|\left(d_m^{(i)}=-1\right)\right.\right) \quad (53)$$

Because there is an equal probability between "0" and "1" in transmitted signal , the equation (53) can be rewritten as

$$\begin{aligned} BER^m_{conventional} &= \frac{1}{2}\left[\Pr ob\left(z_m^{(j)}\le 0\left|\left(d_m^{(i)}=1\right)\right.\right) + \Pr ob\left(z_m^{(j)}\ge 0\left|\left(d_m^{(i)}=-1\right)\right.\right)\right] \\ &= \frac{1}{2}erfc\left(\frac{B\sqrt{P_B}}{\sqrt{2BP_CP_S+BN_o}}\right) \\ &= \frac{1}{2}erfc\left(\sqrt{\frac{BP_B}{2P_CP_S+N_o}}\right) \end{aligned} \quad (54)$$

The previous equation independent on the transmitted m[th] symbol, so the error probability of the m[th] symbol is the same as the BER for the system

$$BER_{conventional}=BER^m_{conventional} = \frac{1}{2}erfc\left(\sqrt{\frac{BP_B}{2P_CP_S+N_o}}\right) \quad (55)$$

The expression (55) gives the analytical BER for the noisy coherent conventional system under the interference of a chaotic signal. It is clear from this expression that the BER for conventional system in combined with chaotic system tends to that of conventional BPSK system when $P_S=0$ (no chaotic signal). Note that at fixed P_C the BER can be improved by:

- Increasing the spreading factor B.
- Increasing the ratio P_B/P_S.

IV. Simulation Example

Suppose as in [6] the chaotic sequence is generated by a map in form of

$$x_{k_C+1}=1-2x_{k_C}^2 \quad (56)$$

The invariant probability density function of x_k, denoted by $\rho(x_{k_C})$ is [6]

$$\rho(x_{k_C})=\begin{cases}\dfrac{1}{\pi\sqrt{1-x_{k_C}^2}}, & \text{if } x_{k_C}\le|1| \\ 0\ , & otherwise\end{cases} \quad (57)$$

So the mean value of x_{k_C} is

$$E[x_{k_C}]=\int_{-\infty}^{\infty}x_{k_C}\rho(x_{k_C})\,dx_{k_C}=\int_{-1}^{1}x_{k_C}\rho(x_{k_C})\,dx_{k_C}=0 \quad (58)$$

$$P_S=E[x_{k_C}^2]=\int_{-\infty}^{\infty}x_{k_C}^2\rho(x_{k_C})\,dx_{k_C}=\int_{-1}^{1}x_{k_C}^2\rho(x_{k_C})\,dx_{k_C}=\frac{1}{2} \quad (59)$$

$$\beta=E[(x_{k_C})^4]=\int_{-1}^{1}x_{k_C}^4\rho(x_{k_C})\,dx_{k_C}=\frac{3}{8} \quad (60)$$

Substitute (59), and (60) in (35) and for simplicity we take $P_C=1$

$$BER_{chaotic}=\frac{1}{2}erfc\left(\frac{\frac{NB}{2}}{\sqrt{\frac{NB}{4}+NBP_B+\frac{NBN_o}{2}}}\right)$$

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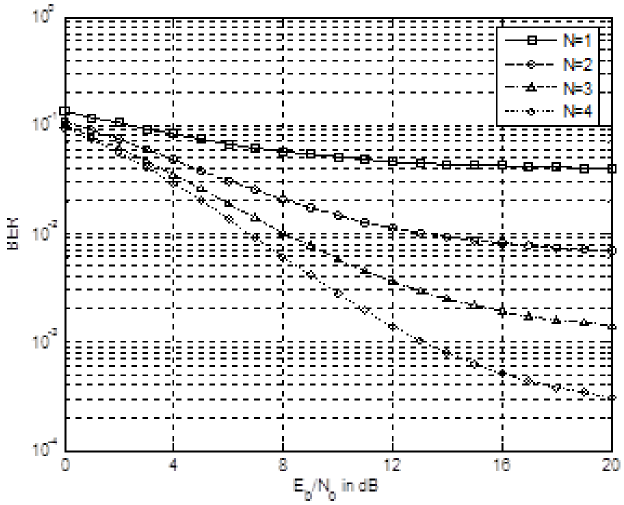

Fig. 4. The BER versus E_b / N_o for coherent chaotic receiver in a combined chaotic-conventional spread spectrum system for different *N* values. The figure is plotted at *B*=100 and P_B / P_s = 15 dB.

$$= \frac{1}{2} erfc\left(\frac{1}{\sqrt{\frac{1}{NB} + \frac{2}{NB}\frac{P_B}{P_S} + \frac{N_o}{E_b}}} \right) \quad (61)$$

Now we will study the behavior of BER in (61) with respect to the variation in three factors, N , B , and P_B/P_S individually. Fig. 4. shows the BER at constant B=100 and constant $P_B/P_S = 15$ dB with variable N (the ratio between the spreading factor of chaotic and PN spreaders). The figure indicates that the BER of chaotic receiver decreases at N increases. That means in order to decrease the BER in coherent chaotic receiver, we should increase the spreading factor of chaotic spreader in the transmitter over the conventional PN spreader.

Fig. 5. shows the effect of spreading factor *B* on the BER. As shown in this figure the BER decreases as *B* increases for constant P_B/P_S and *N*.

In this figure $P_B/P_S = 15$ and *N*=2. The effect of varying P_B/P_S on BER is shown in Fig. 6. where the *B* and *N* are taken constant at 100 and 2 respectively. As expected the BER decreases by decreasing the ratio P_B/P_S or increasing the average power of chaotic signal with respect to the output power from PN spreader.

Now we will extend our study to cover the BER for conventional spread spectrum system in combined chaotic-conventional spread spectrum system. We rewrite (55) as

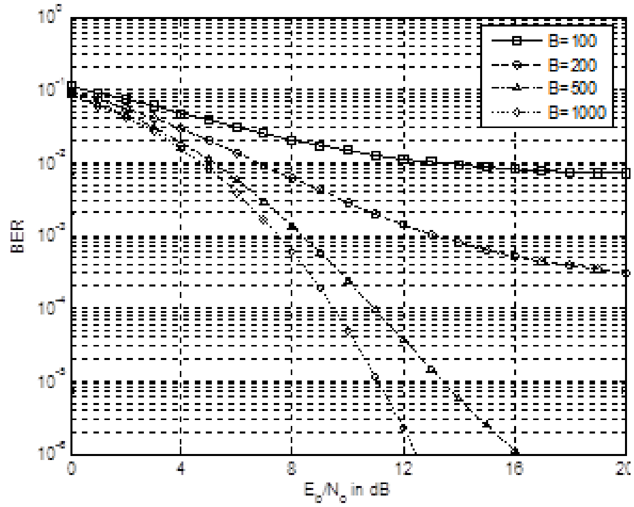

Fig. 5. The BER versus E_b / N_o for coherent chaotic receiver in a combined chaotic-conventional spread spectrum system for different *B* values. The figure is plotted at *N*=2 and P_B / P_s = 15 dB.

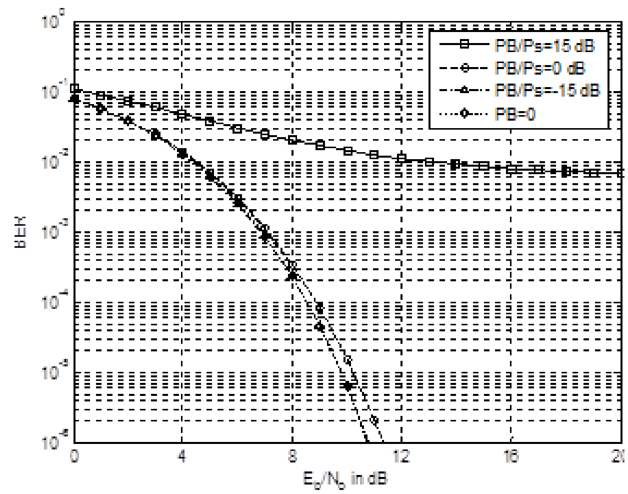

Fig. 6. The BER versus E_b / N_o for coherent chaotic receiver in a combined chaotic-conventional spread spectrum system for different P_B / P_s values. The figure is plotted at *B*=100 and *N*=2. The last two curves are almost identical.

$$BER_{conventional} = \frac{1}{2} erfc\left(\sqrt{\frac{1}{\left(\frac{BP_B}{2P_S}\right)^{-1} + \left(\frac{E_B}{N_o}\right)^{-1}}} \right) \quad (62)$$

To sense the variation of BER, the previous example of chaotic sequence in (56) is taken, Fig. 7. shows the BER for conventional receiver versus the E_B / N_o with varying spreading factor B at constant P_B / P_s = -15 dB.

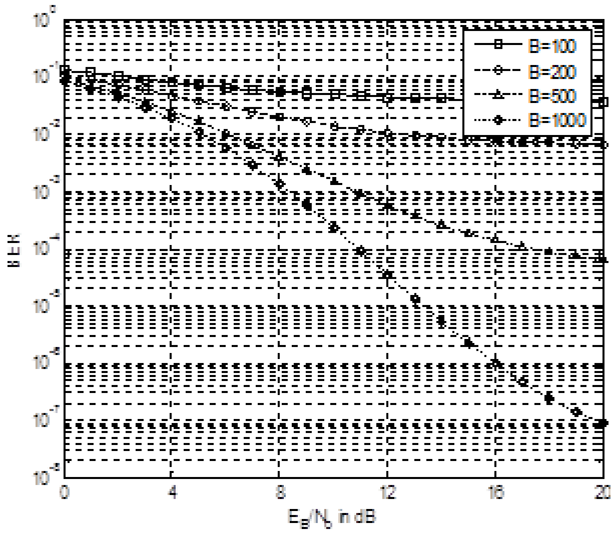

Fig. 7. The BER versus E_B / N_o for coherent conventional receiver in a combined chaotic-conventional spread spectrum system for different *B* values. The figure is plotted at constant P_B / P_s =-15 dB.

As shown in the figure the BER decreases as B increases. Fig. 8. shows BER for conventional receiver versus the E_B / N_o with varying P_B / P_s at constant spreading factor at 100. As expected the BER decreases as the ratio P_B / P_s increases.

V. CONCLUSION

In this work the performance of chaotic-conventional spread spectrum system has been studied. The importance of this study is coming from the variety of conventional spreading systems used nowadays and the desire of enhancement these systems under condition of the original systems are still in used and doesn't degrade with the enhancement action. This paper discusses the system performance under influence of different system parameters and different ratio between the spreading factor of chaotic and PN spreaders. Although robust chaos synchronization techniques are still not available, the great properties of chaos systems are so attractive in communication systems and the results of this paper can indicate the performance of one of various utilities of chaotic systems in communication field.

Appendix

This Appendix presents the analytically proves of the used formulas in coherent chaotic receiver. By starting with (17)

$$E\left[y_m^{(j)} \middle| \left(d_m^{(i)} = +1 \right) \right] = E[A] + E[B] + E[C] \qquad (17)$$

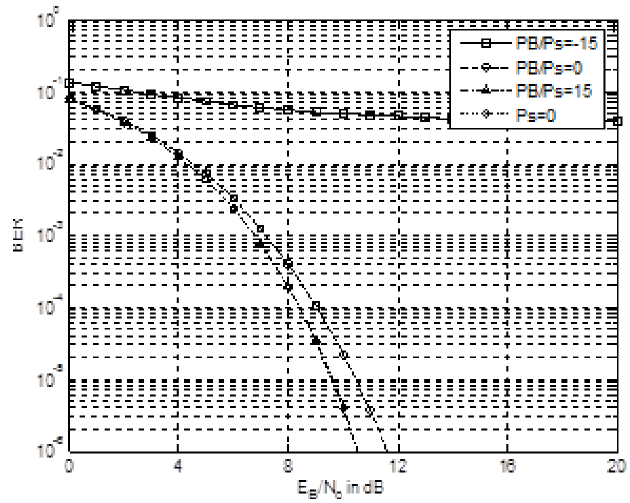

Fig. 8. The BER versus E_B / N_o for coherent conventional receiver in a combined chaotic-conventional spread spectrum system for different P_B / P_s values. The figure is plotted at constant *B*=100. The last two curves are almost identical.

Where

$$A = \sqrt{P_C} \sum_{k_C = NB(m-1)+1}^{NBm} (x_{k_C}^{(i)})^2 \qquad (14)$$

$$B = \sqrt{P_B} \sum_{k_C = NB(m-1)+1}^{NBm} g_{k_P}^{(i)} x_{k_C}^{(i)} \qquad (15)$$

$$C = \sum_{k_C = NB(m-1)+1}^{NBm} n_{k_C} x_{k_C}^{(i)} \qquad (16)$$

And the $\mathrm{var}[X]$ denotes by

$$\mathrm{var}[X] = E[X^2] - E^2[X] \qquad (63)$$

So

$$\mathrm{var}[A]=E\left[P_C\sum_{k_C=NB(m-1)+1}^{NBm}(x_{k_C}^{(i)})^4\right]-E^2\left[\sqrt{P_C}\sum_{k_C=NB(m-1)+1}^{NBm}(x_{k_C}^{(i)})^2\right]$$

$$=P_C NBE\left[(x_{k_C}^{(i)})^4\right]-\sqrt{P_C}NBP_S \qquad (64)$$

And $P_S=E[(x_{k_C}^{(i)})^2]$(the average power of chaotic signal)

$$\mathrm{var}[B]=E\left[P_B\sum_{k_C=NB(m-1)+1}^{NBm}(g_{k_P}^{(i)}x_{k_C}^{(i)})^2\right]-E^2\left[\sqrt{P_B}\sum_{k_C=NB(m-1)+1}^{NBm}g_{k_P}^{(i)}x_{k_C}^{(i)}\right]$$

$$=P_B NBE[(g_{k_P}^{(i)})^2]E[(x_{k_C}^{(i)})^2]$$
$$-NB\sqrt{P_B}E^2[g_{k_P}^{(i)}]E^2[x_{k_C}^{(i)}]$$
$$=P_B NBP_S \qquad (65)$$

$$\mathrm{var}[C]=E\left[\sum_{k_C=NB(m-1)+1}^{NBm}(n_{k_C}x_{k_C}^{(i)})^2\right]-E^2\left[\sum_{k_C=NB(m-1)+1}^{NBm}n_{k_C}x_{k_C}^{(i)}\right] \qquad (66)$$

$$=NB\frac{N_o}{2}P_S$$

And for covariance expressions of coherent chaotic receiver

$$\mathrm{cov}[X,Y]=E[XY]-E[X]E[Y]$$

So

$$\mathrm{cov}[A,B]=E\left[\left(\sqrt{P_C}\sum_{k_C=NB(m-1)+1}^{NBm}(x_{k_C}^{(i)})^2\right)\times\left(\sqrt{P_B}\sum_{k_C=NB(m-1)+1}^{NBm}g_{k_P}^{(i)}x_{k_C}^{(i)}\right)\right]$$

$$-E\left[\sqrt{P_C}\sum_{k_C=NB(m-1)+1}^{NBm}(x_{k_C}^{(i)})^2\right]\times E\left[\sqrt{P_B}\sum_{k_C=NB(m-1)+1}^{NBm}g_{k_P}^{(i)}x_{k_C}^{(i)}\right]$$

$$=\sqrt{P_C P_B}E\left[\sum_{k_C=NB(m-1)+1}^{NBm}(x_{k_C}^{(i)})^3 g_{k_P}^{(i)}\right]-0$$

$$=\sqrt{P_C P_B}\sum_{k_C=NB(m-1)+1}^{NBm}E\left[(x_{k_C}^{(i)})^3\right]\times E\left[g_{k_P}^{(i)}\right]=0 \qquad (67)$$

$$\mathrm{cov}[A,C]=E\left[\left(\sqrt{P_C}\sum_{k_C=NB(m-1)+1}^{NBm}(x_{k_C}^{(i)})^2\right)\times\left(\sum_{k_C=NB(m-1)+1}^{NBm}n_{k_C}x_{k_C}^{(i)}\right)\right]$$

$$-E\left[\sqrt{P_C}\sum_{k_C=NB(m-1)+1}^{NBm}(x_{k_C}^{(i)})^2\right]E\left[\sum_{k_C=NB(m-1)+1}^{NBm}n_{k_C}x_{k_C}^{(i)}\right]$$

$$=\sqrt{P_C}E\left[\sum_{k_C=NB(m-1)+1}^{NBm}n_{k_C}(x_{k_C}^{(i)})^3\right]$$

$$-\left[\left(\sqrt{P_C}\sum_{k_C=NB(m-1)+1}^{NBm}E[(x_{k_C}^{(i)})^2]\right)\times\left(\sum_{k_C=NB(m-1)+1}^{NBm}E[n_{k_C}]E[x_{k_C}^{(i)}]\right)\right]$$

$$=\sqrt{P_C}\sum_{k_C=NB(m-1)+1}^{NBm}E[n_{k_C}]E[(x_{k_C}^{(i)})^3]-0=0 \qquad (68)$$

$$\mathrm{cov}[B,C]=E\left[\left(\sqrt{P_B}\sum_{k_C=NB(m-1)+1}^{NBm}g_{k_P}^{(i)}x_{k_C}^{(i)}\right)\times\left(\sum_{k_C=NB(m-1)+1}^{NBm}n_{k_C}x_{k_C}^{(i)}\right)\right]$$

$$-E\left[\sqrt{P_B}\sum_{k_C=NB(m-1)+1}^{NBm}g_{k_P}^{(i)}x_{k_C}^{(i)}\right]\times E\left[\sum_{k_C=NB(m-1)+1}^{NBm}n_{k_C}x_{k_C}^{(i)}\right]$$

$$=\sqrt{P_B}\sum_{k_C=NB(m-1)+1}^{NBm}E[g_{k_P}^{(i)}]E[n_{k_C}]E[(x_{k_C}^{(i)})^2]$$

$$-\left[\left(\sqrt{P_B}\sum_{k_C=NB(m-1)+1}^{NBm}E[g_{k_P}^{(i)}]E[x_{k_C}^{(i)}]\right)\times\left(\sum_{k_C=NB(m-1)+1}^{NBm}E[n_{k_C}]E[x_{k_C}^{(i)}]\right)\right]$$

$$=0 \qquad (69)$$

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