A Performance Study of Energy Detection Based Spectrum Sensing for Cognitive Radio Networks

James D. Gadze¹, Oyibo, A. Michael², Ajobiewe, N. Damilola³

¹,²,³Department of Electrical/Electronics Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.

Abstract—This study evaluates the performance of the energy detection based spectrum sensing technique in noisy and fading environments. Both single user detection and cooperative detection situations were investigated. Closed form solutions for the probabilities of detection and false alarm were derived. Analytical results were verified by numerical computations using Monte Carlo method in MATLAB. The performance of the energy detection technique was evaluated by use of Receiver Operating Characteristics (ROC) curves over AWGN and fading (Rayleigh & Nakagami-m) channels. Results show that for both single user detection and cooperative detection, the energy detection technique performs better in AWGN than in fading environment. The performance of cooperative detection in fading environment on the other hand, outperforms that of the single user detector.

Keywords—Cognitive Radio, Energy Detection, Spectrum Sensing, Telecommunications, Wireless Communication.

I. INTRODUCTION

By convention, licensed spectrum is allocated over long time periods, and is meant to be used only by licensees. A government agency apportions license for spectrum use; referred to as the Fixed Spectrum Allocation (FSA) scheme. With this, the radio spectrum is split into bands allocated to distinct technology based services, e.g. mobile telephony, radio and TV broadcast on absolute basis. The FSA model guarantees exclusive use of the frequency spectrum by licensed users (i.e. the primary user (PU)) [1].

As a consequence of the transition from regular voice-only communication to multimedia type applications demanding higher data rates, this plan will not have the capacity for emerging applications. Records from the FCC frequency allocation chart depict near occupation of usable spectrum by government and commercial operators, leaving little bandwidth for future wireless systems [2].

However, studies [3] show spectrum use is intense on certain portions while a significant amount remain underutilized. This is due, in part, to the fact that most carriers (i.e. license owners) do not transmit at all times in all geographic locations where the license covers. Records from the FCC indicate spectrum allocated in the bands below 3GHz have a utilization range of 15% to 85% [4].

Presented with the restrictions of the frequency spectrum, ground-breaking techniques that allow exploiting the available spectrum are required. As a result, Dynamic Spectrum Access (DSA) was proposed to solve this inefficiency challenge. With this concept, the licensed radio spectrum is optimized by secondary users that employ opportunistic spectrum access (OSA) of the frequency bands not occupied by the primary or licensed user. The enabling technology for this Next Generation (xG) network is the Cognitive Radio (CR). The Cognitive Radio is an intelligent radio platform with the ability to exploit its environment to increase spectral efficiency and capacity. To this end, the CR technology is envisaged to enable identification, use and management of vacant spectrum, known as spectrum holes or whitespaces [5].

To keep this effect at an acceptable level, secondary users will sense the spectrum to detect whether it is available or not. Hence, reliable spectrum sensing is the most integral function, upon which the entire process of the cognitive radio rests [6]. The challenge then is that the procedure needs to have as little delay as possible so that once channels are available, transmissions should occur right away. Consequently, few false detections would be expected.

There exist three spectrum sensing (SS) techniques in literature. These are the matched filter, cyclostationary feature and energy detection methods. Both the matched filter and cyclostationary feature based detection concern knowledge of a signal, which is not always obtainable in practical scenarios [7]. More so, these two techniques require a significant amount of time to detect a signal; adding complexity in the detection process. Among them, however, energy detection (ED) scheme is widely feasible, since it does not require a priori knowledge of the primary signals and has the least complexity. The ED method essentially ascertains the energy of a received signal to decide whether a detected signal is noise or a primary user's signal [8].

It is noteworthy that a failure in the performance of spectrum sensing implies a missed opportunity for secondary users to utilize the white space of the spectrum causing interference to the primary users [9].
Therefore, a study in this regard is essential, so as to enhance the throughput of secondary users and to protect primary users from unintended interference. Consequently, the energy detection technique fits well with the general purpose of sensing the spectrum for different wireless communication systems. This justifies the intensive study being carried out on the ED technique and the motivation for this paper.

This study aims at assessing the performance of an energy detector for spectrum sensing. Both the case of a single detector and nodes of cooperative detectors will be explored. To do this, use is made of wireless channel models; exemplifying fading and non-fading characteristics.

The rest of this paper is organised as follows. In section II, using a system model, the energy detection technique is assessed. Closed form expressions for the probability of detection and false alarm will be derived respectively. Section III takes on the performance of a single detector over non-fading and fading channels. Section IV provides an assessment of a cooperative node of detectors over both Rayleigh and Nakagami-m fading channels. Concluding remarks will be presented in section V.

Originally, energy detection approach was outlined by Urkowitz in [8], were signals are assumed deterministic, and exact noise variance is known beforehand. Sampling theorem is then applied to estimate the received signal energy and from the chi-square statistics of the resulting sum of the squared Gaussian random variables, signal detection is reduced to a simple identification problem; formalized as a hypothesis test. Though, this archetype energy detector addressed detection of unknown deterministic signals buried in Gaussian noise, the analysis however, concerns the time domain; creating a difficulty when estimating the spectral component.

Since then however, ED analysis has been considered with several modifications in literature. In Shehata et al. [10], an adaptive scheme to explore ED based spectrum sensing is proposed. The model consist of a PU transmitting a QPSK modulated signal within a 200 KHz bandwidth. Sampling frequency is set 8 times the bandwidth and a 1024-point FFT is used to compute the received signal energy. Results suggest execution of spectrum sensing on emergence of the PU in the wake of the sensing time. However, from the choice of bandwidth under consideration, this study is restricted to frequency modulated (FM) signals. Numerical analysis of the ED method over fading channels is presented by Reisi et al. in [11].

In this work, deviating from exact solutions, the authors deduce approximate expressions for the probability of detection ($P_D$) for energy detection over Nakagami fading channels only. Furthermore, expressions relating the number of samples, sensing time to SNR for a given $P_D$ and $P_{FA}$ in this channel were also deduced.

In [12], applying single narrowband energy detector nodes to sense multiple channels is considered. For this, the detector works with a preset factor - based on past primary user action - to ascertain which channels to exploit in the future. This approach proves to reduce sensing time; allaying the initial concern faced by the archetype energy detector.

Shadowing, multipath fading and receiver uncertainty effects pose severe challenges to single user detection approach. Hence the concept of cooperation is studied in [13]. Within this context, each cooperating node employs energy detection locally, while sharing the raw sensed information with other node(s). This is dependent on the cooperative scheme adopted. Common cooperative spectrum sensing (CSS) schemes include centralized, decentralized and relay-assisted arrangement of nodes [14]. Fig. 1 illustrates the cooperative approach to spectrum sensing.

![Fig. 1. Cooperative spectrum sensing](image)

A network of cooperative energy detector nodes to minimise sensing time is carried out in [16]. Total sensing time to be minimized include integration time for local processing and reporting time of an energy detector; proportional to the number of cooperating SU. The results from this work show that, for higher detection sensitivity, a longer integration time is required. This is unlike the general notion of cooperation, wherein an increase in the number of cooperating nodes reduce the required sensing time to achieve the same level of detection sensitivity.
In [17], a theoretic performance study of cooperative energy detector nodes is considered for channels encountering shadowing. The analytical framework consists of data and decision fusion models applied to explore operation of this method. In this analysis however, only Rayleigh fading channel is considered.

While several variants of the ED technique is been propounded in literature for spectrum sensing, it is apparent that not much work has been done in assessing the performance of this scheme as a de facto standard for opportunistic spectrum access towards the realisation of the CR paradigm. Hence, the aim of this study is to assess the performance of the energy detection technique for spectrum sensing. The case of a single detector and nodes of cooperative detectors are explored.

II. SYSTEM MODEL

In implementing energy detection, the received signal \( x(t) \) is filtered by a band pass filter (BPF), followed by a square law device. The band pass filter serves to reduce the noise bandwidth. Thus, noise at the input to the squaring device possesses a band-limited, flat spectral density [8]. The output of the integrator is the energy of the input to the squaring device over the time interval \( T \). Next, the output signal from the integrator (the decision statistic) \( Y \), is

\[
x(t) = \begin{cases} 
  n(t) & H_0 \\
  h * s(t) + n(t) & H_1 
\end{cases}
\]

\( x(t) \) is the received sample signal (or whitespace sample) to be analysed at each instant \( t \), \( n(t) \) is additive noise (having zero-mean and variance \( \sigma^2 \)), \( h \) is the channel gain between the primary signal transmitter and the detector. \( s(t) \) - is the transmitted signal to be detected.

Overall, the goal is to observe the sample signal \( x(t) \), then have some rule decide the correct hypothesis based on the test statistic being either greater or less than the threshold. Characterising the performance of such a decision rule is realised using some metrics.

A. Performance Metrics and Measurement

The correctness of sensing signal availability is defined using quality parameters. This feature make up the performance metrics; among which are the probability of detection \( (P_D) \), false alarm probability \( (P_{FA}) \) and probability of missed detection \( (P_M) \). \( P_D \) specifies that a detector makes a correct detection that a channel is occupied, hence it is an indicator of the level of interference protection provided to the primary user. A large \( P_D \) denotes exact sensing; hence a small chance of interference. An SU misses the chance to exploit a free spectrum when a false alarm event occurs. \( P_{FA} \) should be kept as low as possible in order to prevent underutilization of transmission opportunities. This is an important measure in the study of a spectrum sensing technique. The probability of declaring the spectrum space vacant \( H_0 \), when it is indeed occupied \( H_1 \), is referred to as the probability of missed detection \( (P_M) \). A high \( P_M \) implies an increase in the chance of interference between the PU and the SU. If the detection fails, or a “miss detection” occurs, the SU initiates a transmission, resulting in interference with the PU signal; contravening the opportunistic access concept. In essence, the spectrum sensing method should show a high probability of detection (low miss detection probability) and low probability of false alarm.

To quantify and depict receiver performance, use is made of receiver operating characteristics (ROC) curves. These graphs show relative trade-offs between detection probability and false alarm rates, (i.e. \( P_D \) versus \( P_{FA} \)), thus allowing the determination of an optimal threshold. There assist in exploring the relationship between sensitivity (detection probability) and specificity (false alarm rate). To plot ROC curves, one parameter is varied while the other is fixed.

Using an energy detector, a test statistic is computed from discrete samples of the channel under investigation.

\[
Y = \sum_{k=1}^{M} |x[n]|^2
\]

\( Y \) is the test statistic at the energy detector node; the number of samples under test is \( M \). It is assumed that the noise power at the ED node is normally distributed with zero mean and unity variance.
Thus, the received signal $x(t)$ is normalised with respect to the noise power $[18]$. Though the estimate of this noise power can be said to be uncertain, for this work however, these uncertainties are assumed negligible.

From (2), the distribution of the received signal energy at the ED node is written as,

$$ Y = \left[ \chi^2_{2d} + \chi^2_{2d}(2\gamma) \right] H_o H_t $$

(3)

$\chi^2_{2d}$ and $\chi^2_{2d}(2\gamma)$ are the central and non-central chi-square distributions, respectively, $d$ is the time-bandwidth product at the node, $\gamma$ is the non-centrality parameter equal to the signal to noise ratio $[8]$, i.e. $\gamma = E/N_0$ (4)

The PDF for a chi-squared distribution, $Y$ (from $[19]$) is,

$$ f_y(y) = \begin{cases} \frac{1}{2^d \Gamma(d)} y^{d-1} e^{-\frac{y}{2}} & H_0 \\ \frac{1}{2^d \Gamma(d)} \left(\frac{d}{2}\right)^{\frac{d}{2}} \Gamma\left(\frac{d}{2}, \frac{y}{2}\right) & H_t \end{cases} $$

(5)

$\Gamma(.)$ is the gamma function and $I_v(.)$ is the $v$th-order modified Bessel function of the first kind.

An approximate solution for the $P_D$ and $P_{FA}$ over an AWGN is presented in $[8]$, however this is computationally complex. From eqn. (4), closed-form expressions for both $P_D$ and $P_{FA}$ for an ED over additive white Gaussian noise (AWGN) channel is derived. In this kind of channel, the only impairment is noise with a constant spectral density. The AWGN channel is considered a non-fading channel and it produces insight to the behaviour of a system before other phenomenon is considered.

$P_D$ is the probability that $H_t$ is selected when a signal is present. For a threshold $\kappa$, $P_D$ and $P_{FA}$ is defined as;

$$ P_D = P(Y > \kappa | H_t) $$

(6)

$$ P_{FA} = P(Y > \kappa | H_0) $$

(7)

Expressing $P_D$ and $P_{FA}$ in terms of the PDF yields

$$ P_{FA} = \int_{\kappa}^{\infty} f_y(y) dy $$

(8)

from (4)

$$ \Gamma(d, x) = \int_{x}^{\infty} t^{d-1} e^{-t} dt $$

$$ P_{FA} = \frac{1}{2^d \Gamma(d)} \int_{\kappa}^{\infty} \left(\frac{y}{2}\right)^{d-1} e^{-\frac{y}{2}} dy $$

(9)

Substituting $\frac{\sqrt{y}}{2} = t$ with changed limits, and expressing equation (8) in terms of the gamma function; defined by $\Gamma(d, x) = \int_{x}^{\infty} t^{d-1} e^{-t} dt$, the $P_{FA}$ is described by $[19]$,

$$ P_{FA} = \frac{\Gamma \left( d - \frac{\kappa}{2} \right)}{\Gamma(d)} $$

(10)

It is apparent from eqn. (10) that $P_{FA}$ is dependent on two parameters; the time-bandwidth product, $d$, and threshold value, $\kappa$. Hence, $P_{FA}$ is not related to the SNR.

$P_D$ is obtained from the cumulative distribution function (CDF) of $Y$, (from eqn. (4)) as,

$$ P_D = 1 - F_y(\kappa) $$

(11)

For an even number of degrees of freedom, $2d$ in this case CDF of $Y$ is described by

$$ F_y(y) = 1 - Q_d(\sqrt{\kappa}, \sqrt{y}) $$

(12)

From eqns. (10) & (11), $P_D$ for an energy detector over AWGN channel can be evaluated from;

$$ P_D = Q_d(\sqrt{2\gamma \kappa}) $$

(13)

In real communication systems, signals take more than a path between the transmitter and receiver. Fading distribution models that explore uncertainties encountered in a channel serve as tools for studying multipath and path loss features. Among these are the Rayleigh and Nakagami channel fading models.

By averaging the conditional $P_D$ in the AWGN case, (as given in eqn. (12)) over the SNR fading distribution, closed form expression for the $P_D$ in Rayleigh fading channels is expressed as $[20]$;

$$ P_{aw} = e^{\frac{\kappa^2}{4}} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\kappa^2}{2}\right)^n + \left(1 + \frac{\kappa^2}{4}\right)^{d-1} \left[ e^{\frac{\kappa^2}{4(d-1)}} - e^{\left(\frac{\kappa^2}{4}\right)^{d-1}} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\kappa^2}{2(1+\gamma)}\right)^n \right] $$

(14)

It is noteworthy that the expression for the $P_{FA}$ will remain unchanged under all fading channels, since $P_{FA}$ is independent of SNR (as shown in eqn. (10)).
The Rayleigh fading model considers urban multipath features, especially effects of the ionosphere and troposphere. It describes the statistical time varying nature of the received envelope of a flat fading signal or the envelope of an individual multipath component.

For examining the performance of communication systems over generalized fading channels, an appropriate model is the Nakagami fading distribution. The $P_D$ over Nakagami channel is determined by averaging eqn. (12) over the Nakagami distribution to obtain a closed form expression given as [20]:

$$P_{\text{out}} = \alpha \left[ G_1 + \beta \sum_{n=1}^{\infty} \frac{(\kappa/2)^n}{n!} F_1 \left( m; n+1; \frac{\kappa - \gamma}{2m + \gamma} \right) \right]$$ (15)

Where $m$ is the Nakagami-m fading parameter, describing the severity of fading; $m < 1$ suggests severe fading, while $m > 1$ indicates less severe fading. For the special case of $m=1$ in eqn. (15) gives an alternate expression for $P_{\text{out}}$; numerically equivalent to eqn. (14). $F_1 (\cdot;\cdot;\cdot)$ is the confluent hypergeometric function, whose solution and that of $\beta$ and $G_1$ are provided in [21] section 6.2.

With cooperative spectrum sensing (CSS), sensed information collected at the various locations of the SU energy detector is used for jointly ascertaining spectrum occupancy. This provides diversity gains against channel fading effects. With this, the odds of multiple receivers undergoing fading conditions at the same time is less likely; compared to a situation where only a single detector is employed.

With CSS, $M$ samples of the received signal obtained by $N$ energy detectors in a network is considered. The sensed data is in the form of 1-bit binary decisions sent to the fusion centre (FC) (as shown in Fig. 1). Combining sensed results at the FC to achieve a global result is termed Data fusion [7]. Hard decision combining rule (OR, AND, and MAJORITY) is executed at the FC for this purpose. A large number of $N$ energy detectors will increase the complexity of the detection circuitry. For the analysis of CSS, it is presumed SUs receive the primary signal with the same local mean power, the distance between any two sensing nodes are negligible; hence the noise and average SNR are the same for all SUs.

Assuming uncorrelated decisions for $N$ detectors, applying the $k$-out-of-$N$ decision fusion rule, where a decision is reached once $k$ out of $N$ detectors correspond, the effective detection and false alarm probabilities at the fusion centre is expressed by:

$$P_x^f = \sum_{i=1}^{N} \prod_{j=1}^{i} P_x^{(j)} \prod_{j=i+1}^{N} (1 - P_x^{(j)})$$ (16)

Where ($x=FA$) and ($x=\bar{D}$) represents the $P_{FA}$ and $P_D$ respectively. The case of $k=1$, corresponds to the “OR” decision rule. These defines a case where at least 1 out of $k$ SUs detect a PU signal; thus the overall result declares signal presence. From eqn. (16) for $k=I$;

$$P_x^f (k=1)=1-\prod_{i=1}^{N} (1 - P_x^{(i)})$$ (17)

For $k=N$, is the “AND” rule; which is when all the local decisions sent to the FC is one, resulting in the final decision being one. i.e.

$$P_x^f (k=N)=\prod_{i=1}^{N} P_x^{(i)}$$ (18)

Placing $k=N/2$ represents the MAJORITY decision rule. This happens when half or more of the local decisions sent to the FC is one - resulting in the terminal decision of one. Putting $k=N/2$ in eqn. (15)

$$P_x^f (k=N/2) = \sum_{i=1}^{N} \prod_{j=1}^{i} (P_x^{(j)}) \prod_{j=i+1}^{N} (1 - P_x^{(j)})$$ (19)

In the next section, a simulation of the case scenarios is presented, including interpretation of results obtained from the analysis, which is based on the approach described.

### III. SIMULATION AND NUMERICAL RESULTS

In this section, through simulations, the performance of an energy detector applied to a secondary user for spectrum sensing is evaluated. All simulations in this work is executed on MATLAB (version R2013a). Monte Carlo (MC) method, which is a stochastic technique (based on the use of random numbers) forms the basis of these simulations. Receiver operating characteristics (ROC) curves is used to compare different scenarios described in the previous sections. Each figure consists of both simulation and analytical results; represented by discrete marks and lines respectively.

Fig. 3 illustrates energy detection over a non-fading (AWGN) channel - a case where the form of interference is only noise-, which is an important starting point for considering the single user case. For this, sample size $N=1000$, time-bandwidth product $d=4$, and SNR is from 0-15dB. It is apparent from this plot that numerical and simulation results correspond, corroborating accuracy of the analysis. From this plot, the probability of miss improves rapidly with increasing SNR ($\gamma$).
Approximately a gain of one order of magnitude is achieved when \( \gamma \) increases from 10 dB to 15dB when a node experiences no channel fading effects. This buttresses the point made earlier that an increase in SNR produces greater detection performance over a non-fading channel.

Fig. 3. ROC curves for single user Energy detection over AWGN.

ROC curves over Rayleigh channel using a single detector for average SNR (\( \gamma \)) values of 0-15dB; \( d = 4 \), sample size \( N = 1000 \) is as shown in Fig. 4. From this \( P_M - P_{FA} \) plot, it is remarkable that the slopes are low for \( P_F < 0.1 \), and a 5 dB increase in SNR (i.e. from 10dB to 15dB) has an increase in \( P_M \) (i.e. reduced \( P_D \)) of up to 0.6 times; compared to the \( P_D \) over AWGN.

Fig. 4. ROC curves for single user Energy detection over Rayleigh fading channel.

It is evident that energy detection performed over a Rayleigh channel exhibits a tough detection performance, compared to that of AWGN. This is not far-fetched, since the fading severity is more in a Rayleigh channel compared to the case of AWGN, (which is a case of no fading, shown previously).

Fig. 5. affirms the concept (from eqn. (4)) that for similar signal energy, improved performance is achieved by employing less number of samples; as obtained when the energy of the signal \( E_s \), increases for a given number of samples \( N \).
Next, energy detection over Nakagami-m fading channel is explored. This is as depicted in Fig. 6.

From Fig. 6, the $P_M$ (increased detection performance) rapidly improves with increasing average SNR. A gain in the order of a magnitude is observed for SNR values of 10dB and 15dB respectively; from the position of the $P_M$ for $m=2$, compared to the Rayleigh case of $m=1$ in Fig. 4. This shows greater performance is achieved in a Nakagami fading channel than a Rayleigh case of fading, since fading severity is less (from $m=2$ to $m=1$).

In Fig. 7, using 10 energy detectors, the performance comparison of the various data fusion methods involved in cooperative spectrum sensing (CSS) is considered.
This figure asserts that the OR decision fusion rule exhibits beneficial performance compared to the MAJORITY and AND combining rules. The OR rule involves result of a minimum of a single user out of K energy detector nodes to declare the availability or presence of a PU. Though AND fusion rule indicates a slightly better performance at low $P_{FA}$. Since the OR combining rule minimizes communication overhead, this fusion rule will be adopted in the rest of the analysis for cooperative users in the various fading channel models under consideration.

How does cooperative reception improve performance of the energy detection scheme? This enquiry is explored in Fig. 8 and 9 below. Fig. 8 shows complementary ROC performance curves of energy detection over Rayleigh fading. The number of cooperating nodes, $M=10$, with average SNR values of 0, 5, 10,15dB, and time bandwidth product, $d= 5$. The same parameters are applied to the case of Nakagami fading in Fig. 9.

From these curves, the highest performance gain is obtainable over a Nakagami fading channel case, compared to the Rayleigh fading, with the same parameters considered. Hence, it is evident that cooperating sensing combats performance deterioration of the energy detector at severe fading and shadowing environments.

IV. CONCLUSION

In this work, the performance of an energy detector as related to detecting underutilized or unoccupied spectrum was evaluated. From a theory-based sampling approach, energy detection performance is studied for an unknown signal transmitted over non-fading and fading channels. The detection potential is also quantified when a cooperative scheme of users are employed. This study of spectrum sensing is appropriate as emerging wireless technology require more bandwidth, while the spectrum available is underutilised in space and time.

REFERENCES


