Notes on Fuzzy Graph

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Abstract--In this paper, we study some of the properties of fuzzy graph and prove some results on these. Fuzzy graph is the generalization of the ordinary graph and here fuzzy graph is a simple fuzzy graph. So we introduce a new structure of a fuzzy graph.

Keywords-- Fuzzy subset, Fuzzy relation, Strong fuzzy relation, Fuzzy graph, Fuzzy loop, Fuzzy pseudo graph, Fuzzy spanning subgraph, Fuzzy induced subgraph, Fuzzy underling graph, Level set, Degree of fuzzy vertex, order of the fuzzy graph, size of the fuzzy graph, Fuzzy regular graph, Fuzzy strong graph, Fuzzy complete graph.

I. INTRODUCTION

In 1965, Zadeh [3] introduced the notion of fuzzy set as a method of presenting uncertainty. Since complete information in science and technology is not always available. Thus we need mathematical models to handle various types of systems containing elements of uncertainty. After that Rosenfeld[1] introduced fuzzy graphs. Yeh and Bang[6] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graph. It has numerous applications to problems in computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, etc. Nagoor Gani.A [4, 5] introduced a fuzzy graph and regular fuzzy graph. In this paper we introduce the new structure of a fuzzy graph.

Preliminaries:

1.1 Definition: Let X be any nonempty set. A mapping M: X → [0,1] is called a fuzzy subset of X.

1.2 Example: A fuzzy subset A = { (a, 0.3), (b, 0.4), (c, 0.6) } of a set X = { a, b, c }.

1.3 Definition: Let A and B be any two fuzzy subset of X. We define the following relations and operations:

(i) A⊆B iff A(x)≤ B(x) for all x in X.

(ii) A = B iff A(x) = B(x) for all x in X.

(iii) A∩B iff (A∩B)(x) = min { A(x), B(x) } for all x in X.

(iv) A∪B iff (A∪B)(x) = max { A(x), B(x) } for all x in X.

(v) A^c = 1– A = { ( x, 1– A(x) ) / x∈X }.

1.4 Definition: Let M be a fuzzy subset in a set S, the strongest fuzzy relation on S, that is a fuzzy relation V with respect to M given by V(x,y) = min { M(x), M(y) } for all x and y in S.

1.5 Definition: Let V be any nonempty set, E be any set and f: E→V×V be any function. Then A is a fuzzy subset of V, S is a fuzzy relation on V with respect to A and B is a fuzzy subset of E such that

B(e) ≤ S(x,y) / x∈f⁻¹(y,e).

Then the ordered triple F = (A, B, f) is called a fuzzy graph, where the elements of A are called fuzzy points or fuzzy vertices and the elements of B are called fuzzy lines or fuzzy edges of the fuzzy graph F. If f(e) = (x, y), then the fuzzy points (x, A(x) ), (y, A(y) ) are called fuzzy adjacent points and fuzzy points (x, A(x) ), fuzzy line (e, B(e) ) are called incident with each other. If two district fuzzy lines (e₁, B(e₁) ) and (e₂, B(e₂) ) are incident with a common fuzzy point, then they are called fuzzy adjacent lines.

1.6 Definition: A fuzzy line joining a fuzzy point to itself is called a fuzzy loop.

1.7 Definition: Let F = (A, B, f) be a fuzzy graph. If more than one fuzzy line joining two fuzzy vertices is allowed, then the fuzzy graph F is called a fuzzy pseudo graph.

1.8 Definition: F = (A, B, f) is called a fuzzy simple graph if it has neither fuzzy multiple lines nor fuzzy loops.

1.9 Example: F = (A, B, f), where V= {v₁, v₂, v₃, v₄, v₅ }, E = {a, b, c, d, e, h, g } and f: E→V×V is defined by

f(a) = (v₁, v₂), f(b) = (v₂, v₁), f(c) = (v₂, v₃), f(d) = (v₃, v₁), f(e) = (v₃, v₄), f(h) = (v₄, v₅), f(g) = (v₁, v₅). A fuzzy subset A={ (v₁, 0.3), (v₂, 0.5), (v₃, 0.6), (v₄, 0.7), (v₅, 0.9) } of V.
A fuzzy relation \( S = \{((v_1, v_1), 0.3), ((v_1, v_2), 0.3), ((v_1, v_3), 0.3), ((v_2, v_1), 0.3), ((v_2, v_2), 0.5), ((v_3, v_3), 0.5), ((v_4, v_1), 0.3), ((v_5, v_1), 0.3), ((v_5, v_2), 0.5), ((v_5, v_3), 0.6), ((v_5, v_4), 0.6), ((v_5, v_5), 0.6), ((v_6, v_2), 0.5), ((v_6, v_3), 0.7), ((v_6, v_4), 0.7), ((v_6, v_5), 0.7), ((v_6, v_6), 0.9)\} \) on \( V \) with respect to \( A \) and a fuzzy subset \( B = \{(a, 0.2), (b, 0.4), (c, 0.4), (d, 0.4), (e, 0.5), (h, 0.6), (g, 0.2)\} \) of \( E \).

Let \( C \) be a fuzzy subset of \( V \), the fuzzy subset \( D \) of \( E \) is defined as \( D(e) = C(u) \cap C(v) \cap B(e) \), where \( f(e) = (u, v) \) for all \( e \) in \( E \). Then \( H = (C, D, f) \) is called a fuzzy partial subgraph of \( F \).

1.14 Definition: Let \( F = (A, B, f) \) be a fuzzy graph. Let \( (x, A(x)) \in A \). The fuzzy sub graph of \( F \) obtained by removing the fuzzy point \( (x, A(x)) \) and all the fuzzy lines incident with \( (x, A(x)) \) is called the fuzzy subgraph obtained by the removal of the fuzzy point \( (x, A(x)) \) and is denoted \( F - (x, A(x)) \). Thus if \( F - (x, A(x)) = (C, D, f) \) then \( C = A - \{(x, A(x))\} \) and \( D = \{e, B(e) / (e, B(e)) \in B \text{ and } (x, A(x)) \text{ is not incident with } (e, B(e))\} \). Clearly \( F - (x, A(x)) \) is a fuzzy induced sub graph of \( F \). Let \( (e, B(e)) \in B \) then \( F - (e, B(e)) = (A, D, f) \) is called fuzzy sub graph of \( F \) obtained by the removal of the fuzzy line \( (e, B(e)) \), where \( D = B - \{(e, B(e))\} \). Clearly \( F - (e, B(e)) \) is an fuzzy spanning sub graph of \( F \) which contains all the lines of \( F \) except \( (e, B(e)) \).

1.15 Definition: By deleting from a fuzzy graph \( F \) all fuzzy loops and in each collection of fuzzy multiple edges all fuzzy edge but one fuzzy edge in the collection we obtain a fuzzy simple spanning subgraph \( F \), called fuzzy underlying simple graph of \( F \).

1.16 Example:
Fig. 1.3 A fuzzy subgraph of \( F \)

Fig. 1.4 A fuzzy spanning subgraph of \( F \)

Fig. 1.5 A fuzzy subgraph induced by \( P = \{v_1, v_3, v_4, v_5\} \)

Fig. 1.6 A partial fuzzy subgraph induced by \( C \)

Where \( C(v_1) = 1, C(v_3) = 0.6, C(v_4) = 0.8, C(v_5) = 0.6 \).

Fig. 1.7 \( F - (v_1, 0.7) \)

Fig. 1.8 \( F - (b, 0.6) \)
1.17 **Definition**: Let A be a fuzzy subset of X then the **level subset** or \( \alpha – \text{cut} \) of A is

\[ A_{\alpha} = \{ x \in A \mid A(x) \geq \alpha \} \], where \( \alpha \in [0,1] \).

1.18 **Theorem**: Let \( F = (A, B, f) \) be a fuzzy graph with respect to the set V and E.

Let \( \alpha, \beta \in [0,1] \) and \( \alpha \leq \beta \). Then \( (A_{\beta}, B_{\beta+}, f) \) is a subgraph of \( (A_{\alpha}, B_{\alpha+}, f) \).

**Proof**: It is trivial.

1.19 **Theorem**: Let \( F = (A, B, f) \) be a fuzzy graph with respect to the set V and E, the level subsets \( A_{\alpha} \), \( B_{\alpha} \) of A and B subset of V and E respectively. Then \( F_{\alpha} = (A_{\alpha}, B_{\alpha+}, f) \) is a subgraph of \( G = (V, E, f) \).

**Proof**: It is trivial.

1.20 **Theorem**: Let \( H = (C, D, f) \) be a fuzzy subgraph of \( F = (A, B, f) \) and \( \alpha \in [0,1] \). Then \( H_{\alpha} = (C_{\alpha}, D_{\alpha+}, f) \) is a subgraph of \( F_{\alpha} = (A_{\alpha}, B_{\alpha+}, f) \).

**Proof**: Let \( H = (C, D, f) \) be a fuzzy subgraph of \( F = (A, B, f) \) and \( \alpha \in [0,1] \). Suppose \( u \in C_{\alpha} \Rightarrow C(u) \geq \alpha \Rightarrow A(u) \geq C(u) \Rightarrow A(u) \geq \alpha \Rightarrow u \in A_{\alpha} \). Therefore \( H_{\alpha} \) is a fuzzy subgraph of \( F_{\alpha} \).

1.21 **Definition**: Let A be a fuzzy subset of X. Then the **strong level subset** or **strong \( \alpha \)-cut** of A is \( A_{\alpha^+} = \{ x \in A \mid A(x) \geq \alpha \} \), where \( \alpha \in [0,1] \).

1.22 **Theorem**: Let \( F = (A, B, f) \) be a fuzzy graph with respect to the set V and E. Let \( \alpha, \beta \in [0,1] \) and \( \alpha \leq \beta \). Then \( (A_{\beta+}, B_{\beta+}, f) \) is a subgraph of \( (A_{\alpha+}, B_{\alpha+}, f) \).

**Proof**: It is trivial.

1.23 **Theorem**: Let \( F = (A, B, f) \) be a fuzzy subgraph with respect to the set V and E, the level subsets \( A_{\alpha}, B_{\alpha} \) of A and B subset of V and E respectively. Then \( F_{\alpha} = (A_{\alpha}, B_{\alpha}, f) \) is a subgraph of \( G = (V, E, f) \).

**Proof**: It is trivial.

1.24 **Theorem**: Let \( H = (C, D, f) \) be a fuzzy subgraph of \( F = (A, B, f) \) and \( \alpha \in [0,1] \). Then \( H_{\alpha} = (C_{\alpha}, D_{\alpha+}, f) \) is a subgraph of \( F_{\alpha} = (A_{\alpha}, B_{\alpha+}, f) \).

**Proof**: Let \( H = (C, D, f) \) be a fuzzy subgraph of \( F = (A, B, f) \) and \( \alpha \in [0,1] \). Suppose \( u \in C_{\alpha} \Rightarrow C(u) \geq \alpha \Rightarrow A(u) \geq C(u) \Rightarrow A(u) \geq \alpha \Rightarrow u \in A_{\alpha} \). Therefore \( H_{\alpha} \) is a fuzzy subgraph of \( F_{\alpha} \).

1.25 **Theorem**: Let \( F = (A, B, f) \) be a fuzzy graph with respect to the set V and E, the level subsets \( A_{\alpha} \), \( B_{\alpha} \) of A and B subset of V and E respectively. Then \( F_{\alpha} = (A_{\alpha}, B_{\alpha}, f) \) is a subgraph of \( G = (V, E, f) \).

**Proof**: Since \( A_{\alpha} \) and \( B_{\alpha} \) are subset of V. Clearly \( F_{\alpha} \) is a subgraph of \( G \). Also \( F_{\alpha} \cup F_{\beta} \) is a subgraph of \( G \).

1.26 **Definition**: Let \( F = (A, B, f) \) be a fuzzy graph. Then the **degree of a fuzzy vertex** is defined by \( d(v) = \sum_{e \in f^{-1}(v)} B(e) \). Let \( e \in D_{\alpha} \Rightarrow D(e) \geq \alpha \Rightarrow B(e) \geq D(e) \geq \alpha \Rightarrow B(e) \geq \alpha \). Therefore \( H_{\alpha} \) is a fuzzy subgraph of \( F_{\alpha} \).

1.27 **Definition**: The **minimum degree** of the fuzzy graph \( F = (A, B, f) \) is \( \delta(F) = \wedge \{ d(v) \mid v \in V \} \) and the **maximum degree** of \( F \) is \( \Delta(F) = \vee \{ d(v) \mid v \in V \} \).

1.28 **Definition**: Let \( F = (A, B, f) \) be a fuzzy graph. Then the **order of fuzzy graph** \( F \) is defined to be \( \alpha(F) = \sum_{v \in V} A(V) \).

1.29 **Definition**: Let \( F = (A, B, f) \) be a fuzzy graph. Then the **size of the fuzzy graph** \( F \) is defined to be \( S(F) = \sum_{e \in f^{-1}(x,y)} B(e) \).
1.30 Example:

Here \( d(u) = 1, d(v) = 1.7, d(w) = 1.4, d(x) = 1.3, \delta(F) = 1, \Delta(F) = 1.7, \alpha(F) = 2.5, S(F) = 2.7 \).

1.31 Remark: 0 \leq \delta(F) \leq \Delta(F).

1.32 Remark: Degree of membership value of fuzzy vertices \( v \) need not be equal to the sum of the membership values of all edges incident with fuzzy vertex \( v \).

1.33 Theorem: The sum of the degree of all fuzzy vertices in a fuzzy graph is equal to twice the sum of the membership value of all fuzzy edges. i.e.,

\[
\sum_{v \in V} d(v) = 2S(F).
\]

Proof: Let \( F = (A, B, f) \) be a fuzzy graph with respect to the set \( V \) and \( E \). Since degree of a fuzzy vertex denote sum of the membership values of all fuzzy edges incident on it. Each fuzzy edge of \( F \) is incident with two fuzzy vertices. Hence membership value of each fuzzy edge contributes two to the sum of degrees of fuzzy vertices. Hence the sum of the degree of all fuzzy vertices in a fuzzy graph is equal to twice the sum of the membership value of all fuzzy edges. i.e.,

\[
\sum_{v \in V} d(v) = 2S(F).
\]

1.34 Theorem: Let \( F \) be any fuzzy graph and \( P \) be the number of fuzzy vertices. Then

\[
\frac{2S(F)}{P} \leq \Delta(F).
\]

Proof: Suppose \( F = (A, B, f) \) any fuzzy graph with \( P \) vertices. If every fuzzy vertex has degree \( \delta \) then

\[
\sum_{v \in V} d(v) = \sum_{v \in V} \delta = P\delta.
\]

If every fuzzy vertex has degree \( \Delta \) then

\[
\sum_{v \in V} d(v) = \sum_{v \in V} \Delta = P\Delta.
\]

But \( \sum_{v \in V} \delta \leq \sum_{v \in V} d(v) \leq \sum_{v \in V} \Delta \Rightarrow P\delta \leq 2S(F) \leq P\Delta. \) Hence \( \delta(F) \leq \frac{2S(F)}{P} \leq \Delta(F). \)

1.35 Theorem: Let \( F = (A, B, f) \) be a fuzzy graph with number of fuzzy vertices \( n \), all of whose fuzzy vertices have degree \( s \) or \( t \). If \( F \) has \( p \)-fuzzy vertices of degree \( s \) and \( n-p \) fuzzy vertices of degree \( t \) then

\[
S(F) = \frac{p(s-t) + nt}{2}.
\]

Proof: Let \( V_1 \) be the set of all fuzzy vertices with degree \( s \). Let \( V_2 \) be the set of all fuzzy vertices with degree \( t \). Then

\[
\sum_{v \in V} d(v) = \sum_{v \in V_1} d(v) + \sum_{v \in V_2} d(v)
\]

which implies that \( 2S(F) = ps + (n-p)t \) which implies that \( S(F) = \frac{p(s-t) + nt}{2} \).

II. REGULAR FUZZY GRAPH

2.1 Definition: A fuzzy graph \( F = (A, B, f) \) is called fuzzy regular graph if \( d(v) = k \) for all \( v \) in \( V \).

2.2. Remark: \( F \) is a fuzzy \( k \)-regular graph if and only if \( \delta = \Delta = k \).

2.3 Example:

\[
\text{Fig 2.1
\[\begin{align*}
\text{Here } d(v_1) &= 1.2, d(v_2) = 1.2, d(v_3) = 1.2, \delta = 1.2, \\
\Delta &= 1.2. \text{ Clearly it is a fuzzy 1.2-regular graph.}
\end{align*}\]
\]

2.4 Definition: A fuzzy graph \( F = (A, B, f) \) is called a fuzzy complete graph if every pair of distinct fuzzy vertices are fuzzy adjacent and \( B(e) = S(x, y) \) for all \( x, y \) in \( V \).
2.5 Definition: A fuzzy graph F = (A, B, f) is a fuzzy strong graph if $B(e) = S(x, y)$ for all $e$ in $E$.

2.6 Example:

![Fig. 2.2 A fuzzy strong graph](image1)

2.7 Remark: Every fuzzy complete graph is a fuzzy strong graph. A fuzzy strong graph need not be fuzzy complete graph from the Fig. 2.2.

2.8 Theorem: If F is a fuzzy k-regular graph with p-fuzzy vertices. Then $S(F) = \frac{p^k}{2}$

Proof: Given that the fuzzy graph is a fuzzy k-regular graph, so $d(v) = k$ for all $v$ in $V$. Here there are p-fuzzy vertices, so $\sum_{v \in V} d(v) = \sum_{v \in V} k = pk$ which implies that $S(F) = \frac{p^k}{2}$.

2.9 Remark: In a crisp graph theory any complete graph is regular. But in this fuzzy graph, every fuzzy complete graph need not be fuzzy regular graph. In Fig. 2.3, it is a fuzzy complete graph but not a fuzzy regular graph since $d(v_1) = 0.6$, $d(v_2) = 0.6$, $d(v_3) = 0.4$.

2.10 Theorem: Let F = (A, B, f) be fuzzy complete graph and A is constant function. Then F is a fuzzy regular graph.

Proof: Since A is a constant function, so $A(v) = k$ (say) for all $v$ in $V$ and F is a fuzzy complete graph, so $B(e) = S(x, y)$ for all $x$ and $y$ in $V$ and $x \neq y$.

Therefore membership value of all fuzzy edges are k. Hence $d(v) = (p - 1)k$ for all $v$ in $V$.

2.11 Remark: Let F = (A, B, f) be fuzzy complete graph with p-fuzzy vertices and $A(v) = k$ for all $v$ in $V$. Then F is a fuzzy (p-1)k-regular graph.

2.12 Theorem: If F = (A, B, f) is fuzzy complete graph with p-fuzzy vertices and A is constant function then sum of the membership values of all fuzzy edges is $\frac{p(p-1)}{2}A(v)$ for all $v$ in $V$. i.e., $S(F) = \frac{pC_2 A(v)}{2}$ for all $v$ in $V$.

Proof: Suppose F is a fuzzy complete graph and A is a constant function.

Let $A(v) = k$ for all $v$ in $V$ and $d(v) = (p-1)k$ for all $v$ in $V$. Then $\sum_{v \in V} d(v) = \sum_{v \in V} (p-1)k$ which implies that $2S(F) = p(p - 1)k$. Hence $S(F) = \frac{p(p-1)}{2}k$.

i.e., $S(F) = \frac{pC_2 A(v)}{2}$ for all $v$ in $V$.

2.13 Definition: Let F = (A, B, f) be a fuzzy graph. The total degree of fuzzy vertex $v$ is defined by $D_f(v) = \sum_{e \in \Gamma(v)} B(e) + 2 \sum_{e \in \Gamma(v)} A(v) = d(v) + A(v)$ for all $v$ in $V$.

2.14 Definition: A fuzzy graph F is fuzzy k-totally regular graph if each vertex of F has the same total degree k.

2.15 Example:

![Fig. 2.4](image2)
Here $d_T(v_1) = 1.8$, $d_T(v_2) = 1.8$, $d_T(v_3) = 1.8$, it is fuzzy 1.8-totally regular graph.

2.16 Example: Fig 2.1 it is a fuzzy regular graph, but it is not a fuzzy totally regular graph since $d_T(v_1) = 1.8$, $d_T(v_2) = 2$ and $d_T(v_1) \neq d_T(v_2)$.

2.17 Example: Fig 2.4, it is a fuzzy totally regular graph but it is not a fuzzy regular graph since $d(v_1) = 1.1$, $d(v_3) = 1.2$ and $d(v_1) \neq d(v_3)$.

2.18 Example: Fig 2.5
Here $d(v_i) = 1.2$ for all $i$, $d_T(v_i) = 2$ for all $i$. It is both fuzzy regular graph and fuzzy totally regular graph.

2.19 Example: Fig 2.6
Here $d(v_1) = 1.1$, $d(v_2) = 1.2$, $d(v_3) = .7$, $d_T(v_1) = 1.7$, $d_T(v_2) = 1.9$, $d_T(v_3) = 1$, it is neither fuzzy regular graph nor fuzzy totally regular graph.

2.20 Theorem: Let $F = (A, B, f)$ be fuzzy complete graph and $A$ is constant function. Then $F$ is a fuzzy totally regular graph.

Proof: By theorem 2.10, clearly $F$ is fuzzy regular graph. i.e., $d(v) = (p-1)k$ for all $v$ in $V$. Also given $A$ is constant function. i.e., $A(v) = k$ for all $v$ in $V$. Then $d_T(v) = d(v) + A(v) = (p-1)k + k = pk - k + k = pk$ for all $v$ in $V$. Hence $F$ is fuzzy totally regular graph.

2.21 Remark: Let $F$ be a fuzzy complete graph with $p$-fuzzy vertices and $A(v) = k$ for all $v$ in $V$. Then $F$ is a fuzzy $p(k)$-regular graph.

2.22 Theorem: Let $F = (A, B, f)$ be a fuzzy regular graph. Then $H = (C, B, f)$ is a fuzzy totally regular graph if $C(v_i) = \sum_{i=1}^{n} A(v_i)$ for all $v_i$ in $V$.

Proof: Assume that $F = (A, B, f)$ is a fuzzy $k$-regular graph. i.e., $d(v) = k$ for all $v$ in $V$. Given $C(v_i) = \sum_{i=1}^{n} A(v_i)$ for all $v_i$ in $V$. Then $C(v_i) = k_1$ (say) for all $v_i$ in $V$ and $d_T(v_i) = d(v_i) + C(v_i) = k + k_1$ for all $v_i$ in $V$. Hence $H$ is fuzzy totally regular graph.

2.23 Theorem: Let $F = (A, B, f)$ be a fuzzy regular graph and $A$ is a constant function (i.e., $A(v) = c$ (say) for all $v \in V$). Then $F$ is fuzzy $k$-regular graph if and only if $F$ is fuzzy $(k+c)$-totally regular graph.

Proof: Assume that $F$ is a fuzzy $k$-regular graph and $A(v) = c$ for all $v$ in $V$, so $d(v) = k$ for all $v$ in $V$. Then $d_T(v) = d(v) + A(v) = k + c$ for all $v$ in $V$. Hence $F$ is fuzzy $(k+c)$-totally regular graph. Conversely, Assume that $F$ is fuzzy $(k+c)$-totally regular graph. i.e., $d_T(v) = k+c$ for all $v$ in $V$ which implies that $d(v) + A(v) = k+c$ for all $v$ in $V$ implies that $A(v) = c$ for all $v$ in $V$ implies that $d(v) + c = k+c$ for all $v$ in $V$. Therefore $d(v) = k$ for all $v$ in $V$. Hence $F$ is fuzzy $k$-regular graph.

2.24 Theorem: If $F = (A, B, f)$ is both fuzzy regular graph and fuzzy totally regular graph then $A$ is a constant function.

Proof: Assume that $F$ is a both fuzzy regular graph and fuzzy totally regular graph. Suppose that $A$ is not constant function. Then $A(u) \neq A(v)$ for some $u$, $v$ in $V$. Since $F$ is a fuzzy $k$-regular graph. Then $d(u) = d(v) = k$. Then $d_T(u) \neq d_T(v)$ which is a contradiction to our assumption. Hence $A$ is a constant function.

2.25 Remark: Converse of the above theorem need not be true.
Here \( A(v_i) = 0.4 \) for all \( i \), \( d(v_1) = 1.1 \), \( d(v_2) = 1 \), \( d(v_3) = 0.9 \), \( d_T(v_1) = 1.5 \), \( d_T(v_2) = 1.4 \), \( d_T(v_3) = 1.3 \).

Hence \( F \) is neither fuzzy regular graph nor fuzzy totally regular graph.

2.26 Theorem: If \( F = (A, B, f) \) is a fuzzy c-totally regular graph with \( p \)-fuzzy vertices. Then

\[
S(F) = \frac{pc - o(F)}{2}.
\]

Proof: Assume that \( F \) is a fuzzy c-totally regular graph with \( p \)-fuzzy vertices.

Then \( d_T(v) = c \) for all \( v \) in \( V \) implies that \( d(v) + A(v) = c \) for all \( v \) in \( V \) which implies that \( \sum d(v) + \sum A(v) = \sum c \) for all \( v \) in \( V \) which implies that \( 2S(F) + o(F) = pc \). Hence

\[
S(F) = \frac{pc - o(F)}{2}.
\]

2.27 Theorem: If \( F = (A, B, f) \) is both fuzzy k-regular graph and fuzzy c-totally regular graph with \( p \)-fuzzy vertices. Then \( o(F) = p(c-k) \).

Proof: Assume that \( F \) is fuzzy k-regular graph with \( p \)-fuzzy vertices. Then \( 2S(F) = pk \). By theorem 2.26, \( 2S(F) + o(F) = pc \) implies that \( o(F) = p(c-k) \).

REFERENCES


