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Several Treasures of the Queen of Mathematics

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Abstract
A new method of solving the equation $g = z^2 - y^2$ with given g.
The proper proof of the Fermat's Last Theorem (FLT).
Two complete proofs of the Beal's Conjecture.
The proof of the Erdős-Straus Conjecture.
The proof of the Jeśmanowicz's Conjecture.
Disproof the Oesterlé–Masser Conjecture (the ABC Conjecture).
The short proof of the Goldbach's Conjecture.
MSC: Primary: 11A41, 11D41, 11D45, 11P32; Secondary: 11D61, 11D75, 11D85.

Keywords-- ABC Conjecture, Algebra of Sets, Common Prime Factor, Diophantine Equations, Diophantine Inequalities, Exponential Equations, Fermat Equation, Goldbach Conjecture, Greatest Common Divisor, Newton Binomial Formula, Trinomial Square.

Dedicatory-- **I Dedicate this to My Wife**

I. Introduction

The Theorem 2 (the new method) is dated 03 and 04 June 1997.

The cover of this issue of the Bulletin is the frontispiece to a volume of Samuel de Fermat's 1670 edition of Bachet's Latin translation of Diophantus's Arithmetica. This edition includes the marginalia of the editor's father, Pierre de Fermat. Among these notes one finds the elder Fermat's extraordinary comment in connection with the Pythagorean equation $x^2 + y^2 = z^2$ the marginal comment that hints at the existence of a proof (a demonstratio sane mirabilis) of what has come to be known as Fermat's Last Theorem. Diophantus's work had fired the imagination of the Italian Renaissance mathematician Rafael Bombelli, as it inspired Fermat a century later. [6]

Problem II.8 of the Diophantus's Arithmetica asks how a given square number is split into two other squares. Diophantus's shows how to solve this sum-of-squares problem for $k = 4$ and $u = 2$ [13], inasmuch as for all $k, u \in \{... - 2, -1, 0, 1, 2, ...\}$:

$$k^2 = \left(\frac{2ku}{u^2+1}\right)^2 + \left[\frac{k(u^2-1)}{u^2+1}\right]^2. \quad [2]$$

Thus we get the Diophantus Equation – For all relatively prime natural numbers u, v such that $u - v \in \{1,3,5,...\}$:

$$(u^2+v^2)^2 = u^4 - 2u^2v^2 + v^4 + 4u^2v^2 = (u^2-v^2)^2 + (2uv)^2.$$

We have the primitive Pythagorean triple $(u^2 - v^2, 2uv, u^2 + v^2)$ – the primitive triple, inasmuch as the numbers $u^2 - v^2, 2uv$ and $u^2 + v^2$ are co-prime.

Around 1637, Fermat wrote his Last Theorem in the margin of his copy of the Arithmetica next to Diophantus sum-of-squares problem: it is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain. In number theory, Fermat's Last Theorem (FLT) states that no three positive integers A, B and C can satisfy the equation $A^n + B^n = C^n$ for any integer value of n greater than two. [13]

It is easy to see that if $A^n + B^n = C^n$ then either A, B and C are co-prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pairwise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor). You could then restate FLT by saying that $A^n + B^n = C^n$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases). [10]

Beal has formulated a conjecture in number theory on which he has been working for several years. Let A, B, C, x, y and z be positive integers with $x; y; z > 2$. If $A^x + B^y = C^z$ then A, B and C have a common factor. [5] Beal's conjecture is a generalization of Fermat's Last Theorem. It states: If $A^x + B^y = C^z$, where A, B, C, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a common prime factor. [9] Or,… - below - slightly restated.

The Erdős-Straus Conjecture concerns the Diophantine Equations. The type of Diophantine equation discussed in this paper concerns Egyptian fractions, which deal with the representation of rational numbers as the sum of three unit fractions. [3]

The Jeśmanowicz's Conjecture [1] concerns a pythagorean triples, that is - the Diophantus Equation.

The ABC Conjecture concerns the equation $a + b = c$.

The Goldbach's Conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states: Every even integer greater than 2 can be expressed as the sum of two primes. [14]

II. THE NEW METHOD OF SOLVING THE EQUATION $g = z^2 - y^2$ WITH GIVEN g.

Theorem 1. Let u and v be two relatively prime natural numbers such that $u - v$ is positive and odd. Then $(u^2 - v^2, 2uv, u^2 + v^2)$ is a primitive Pythagorean triple, and each primitive Pythagorean triple arises in this way for some u, v [4], that is to say for all primitive Pythagorean triple there exists different and only one shared pair (u, v).

Theorem 2. For each $g \in \{8,12,16, \dots\}$ or for each $g \in \{3,5,7, \dots\}$ there exist finitely many pairs (s, t) of positive integers such that:

$$g = \left(\frac{g + d^2}{2d}\right)^2 - \left(\frac{g - d^2}{2d}\right)^2 = s^2 - t^2 = (s + t)(s - t) = \frac{g}{d}(s - t) = \frac{g}{d} d = g,$$

where $d|g$ and $d < \sqrt{g}$ and - $d, \frac{g}{d} \in \{2,4,6, \dots\}$ with even g or $d \in \{1,3,5, \dots\}$ with odd g.

Proof of The Main Theorem. For each $g \in \{8,12,16, \dots\}$ or for each $g \in \{3,5,7, \dots\}$ there exist finitely many of divisors d of fixed g such that $d < \sqrt{g}$ and $d, \frac{g}{d} \in \{2,4,6, \dots\}$ with even g, or $d \in \{1,3,5, \dots\}$ with odd g:

$$\left(\frac{g + d^2}{2d} = s \wedge \frac{g - d^2}{2d} = t = s - d \wedge \frac{g}{d} = s + t \wedge d = s - t\right).$$

The number of the pairs (s, t) is finite because $d|g$ with fixed g. This is the proof.

III. THE PROPER PROOF OF THE FERMAT'S LAST THEOREM

Theorem 3 (Femat Last Theorem). *For all* $n \in \{3,4,5, \dots\}$ *and for all* $A, B, C \in \{1,2,3, \dots\}$:

$$A^n + B^n \neq C^n.$$

Proof. Suppose that for some $n \in \{3,4,5, \dots\}$ and for some $A, B, C \in \{1,2,3, \dots\}$:

$$(A^n + B^n = C^n \wedge A + B > C \wedge A^2 + B^2 > C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} > C^{n-1}).$$

In the another case we will have –

For some $n \in \{3,4,5, \dots\}$ and for some $A, B, C \in \{1,2,3, \dots\}$:

$$(A^n + B^n = C^n \wedge A + B \leq C \wedge A^2 + B^2 \leq C^2 \wedge \dots \wedge A^{n-1} + B^{n-1} \leq C^{n-1}) \Longrightarrow$$

$$A^n + B^n < C^n,$$

which is inconsistent with $A^n + B^n = C^n$.

We assume that A, B and C are co-prime. Then only one number out of a solutions $[A, B, C]$ is even. Thus we can assume that $A, C - B \in \{1,3,5, \dots\}$.

Every even number which is not the power of the number 2 has odd prime divisor, hence sufficient that we prove FLT for $n = 4$ and for odd prime numbers $n \in \mathbb{P}$. Fermat did not proved his own theorem for $n = 4$. [8].

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Let

$$\{(2a+b)b: a \in [0,1,2,\dots] \wedge b \in [3,5,7,\dots]\} = \{9,15,21,25,27,33,35,39,45,49,\dots\} \wedge$$
$$\{3,5,7,\dots\} - \{9,15,21,25,27,33,35,39,45,49,\dots\} = \{3,5,7,11,13,17,19,23,29,31,\dots\} = \mathbb{P}.$$

A. Proof For $n = 4$.

For some $C, A, C-B \in \{1,3,5,\dots\}$ and for some $v \in \{1,2,3,\dots\}$ and for some $B \in \{2,4,6,\dots\}$ such that C, A and B are co-prime:

$$\Big\{[A+B-C = 2v \wedge A^2+B^2 > C^2 \wedge C-B+2v = A \wedge C-A+2v = B \wedge (C-B)+(C-A)+2v$$
$$= C \wedge (C-A+2v)^4 = (C-A+A)^4 - A^4 \wedge (C-B+2v)^4 = (C-B+B)^4 - B^4]$$
$$\Rightarrow \Big[(C-A)^2 2v + \frac{3}{2}(C-A)(2v)^2 + (2v)^3 + \frac{4v^4}{C-A}$$
$$= (C-A)^2 A + \frac{3}{2}(C-A)A^2 + A^3 \wedge (C-B)^2 2v + \frac{3}{2}(C-B)(2v)^2 + (2v)^3 + \frac{4v^4}{C-B}$$
$$= (C-B)^2 B + \frac{3}{2}(C-B)B^2 + B^3 \wedge 2v^2 > (C-A)(C-B)\Big]\Big\}.$$

Thus the numbers

$$\frac{4v^4}{C-A}, \frac{4v^4}{C-B} \in \{1,3,5,\dots\}.$$

Without loss for this proof we can assume that $4 \nmid B$.

Hence – For some $c, d, v \in \{1,3,5,\dots\}$ such that c, d are co-prime:

$$(c^4 = C-B \wedge 4d^4 = C-A \wedge 4d^4 + 2v = B \wedge v^2 > 2c^4 d^4) \Rightarrow v > cd.$$

Therefore – For some $c, d, e \in \{1,3,5,\dots\}$ such that c, d and e are co-prime: $cde = v$.

Further it mus be – For some $c, d, e, A \in \{1,3,5,\dots\}$ such that c, d and e are co-prime:

$$(2cde + 4d^4)^4 = [(4d^4+A)^2]^2 - (A^2)^2 = [(4d^4+A)^2 + A^2](2d^4+A)8d^4 \Rightarrow$$
$$2(ce+2d^3)^4 = [(4d^4+A)^2]^2 - (A^2)^2 = [(4d^4+A)^2 + A^2](2d^4+A).$$

We assume that for some $z, w, x \in \{1,3,5,\dots\}$ and for some $y \in \{6,10,14,\dots\}$ such that z, w and x are co-prime:

$$[zw = ce + 2d^3 \wedge x + y = 2d^4 + A + 2d^4 \wedge x = 2d^4 + A \wedge y = 2d^4 \wedge 4 \nmid y \wedge 2(zw)^4 = ((x+y)^2 + (x-y)^2)x$$
$$= 2(x^2+y^2)x \wedge z^4 w^4 = (x^2+y^2)x \wedge (z^2)^2 = (x^2+y^2) \wedge w^4 = x] \Rightarrow 4|y,$$

Which is inconsistent with $4 \nmid y$.

This is the proper proof for $n = 4$.

Remark 1. $(A^2)^2 + (B^2)^2 = (C^2)^2 \equiv 0$ because $2uv(u^2+v^2)\sqrt{2} = B$, so $B \notin \{6,10,14,\dots\}$. On the strenght of the Theorem 1, $[(u^2+v^2)^2 - (2uv)^2 = (u^2-v^2)^2 = A^2 \wedge (u^2+v^2)^2 + (2uv)^2 = C^2] \equiv 0$. On the strength of Theorem 2, $[u^2+v^2 = (uv)^2 - 1 \vee u^2+v^2 = u^2-v^2] \equiv 0$.

This is the remark 1.

B. Proof For $n \in \mathbb{P}$. General Deductions. We assumue that $n \mid A$ in view of [3].

Without loss for this proof we can assume that $4 \nmid B$ or $4 \nmid C$.

Hence – For some $n \in \mathbb{P}$ and for some $C, B \in \{1,2,3,\dots\}$ and for some $C-B, A, v \in \{1,3,5,\dots\}$ such that B, C and A are co-prime:

$$[C-B+2v = A \wedge C-A+2v = B \wedge A+B-2v = C \wedge (C-B+2v)^n = (C-B+B)^n - B^n \wedge (C-A+2v)^n$$
$$= (C-A+A)^n - A^n \wedge (A+B-B)^n + B^n = (A+B-2v)^n].$$

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Thus – For some $n \in \mathbb{P}$ and for some $C, B \in \{1,2,3,\dots\}$ and for some $C-B, A, v \in \{1,3,5,\dots\}$ such that B, C and A are co-prime:

$$\begin{aligned}\Big\{&(C-B)^{n-2}v+(n-1)(C-B)^{n-3}\,v^2+\cdots+2^{n-2}v^{n-1}+\frac{2^{n-1}v^n}{n(C-B)}\\ &=\frac{B}{2}\left[(C-B)^{n-2}+\frac{n-1}{2}(C-B)^{n-3}B+\cdots+B^{n-2}\right]\wedge(C-A)^{n-2}2v+\frac{n-1}{2}(C-A)^{n-3}\,(2v)^2+\cdots\\ &+(2v)^{n-1}+\frac{(2v)^n}{n(C-A)}\\ &=A\left[(C-A)^{n-2}+\frac{n-1}{2}(C-A)^{n-3}A+\cdots+A^{n-2}\right]\wedge(A+B)^{n-2}(-B)+\frac{n-1}{2}(A+B)^{n-3}\,(-B)^2\\ &+\cdots+(-B)^{n-1}=(A+B)^{n-2}(-2v)+\frac{n-1}{2}(A+B)^{n-3}\,(-2v)^2+\cdots+(-2v)^{n-1}+\frac{(-2v)^n}{n(A+B)}\Big\}.[3]\end{aligned}$$

Therefore – For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5,\dots\}$ such that n, e, m, c and h are co-prime:

$$[nemch = v \wedge n \nmid emch \;\wedge h^n = C - A \wedge n^{n-1}c^n = C - B].$$

$\boldsymbol{B.1.}$ *Proof For Odd* $A, B, C - B$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5,\dots\}$ such that n, e, m, c and h are co-prime:

$$\begin{gathered}[n^{n-1}c^n + 2nemch = A \wedge h^n + 2nemch = B \wedge 2^n m^n = A + B = n^{n-1}c^n + h^n + 4nemch \wedge n^{n-1}c^n + B = C]\\ \Longrightarrow [2^n m^n - h^n = n^{n-1}c^n + 4nemch \;\wedge n \mid 2m - h \wedge n^2 \mid 2^n m^n - h^n] \Longrightarrow n \mid emch,\end{gathered}$$

which is inconsistent with $n \nmid emch$. ♠

$\boldsymbol{B.2.}$ *Proof For Even* $B, C - A$.

For some $n \in \mathbb{P}$ and for some $e, m, c, h \in \{1,3,5,\dots\}$ such that n, e, m, c and h are co-prime:

$$\begin{gathered}[n^{n-1}c^n + 2nemch = A \wedge 2^n h^n + 2nemch = B \wedge m^n = A + B = n^{n-1}c^n + 2^n h^n + 4nemch \wedge n^{n-1}c^n + B = C]\\ \Longrightarrow [m^n - 2^n h^n = n^{n-1}c^n + 4nemch \;\wedge\; n \mid m - 2h \;\wedge\; n^2 \mid m^n - 2^n h^n] \Longrightarrow n \mid emch,\end{gathered}$$

which is inconsistent with $n \nmid emch$. ♠ This is the proper proof for $n \in \mathbb{P}$. This is the proof.

Remark 2. For some $n \in \mathbb{P}$ and for some $p, q, w, r, x \in \{1,3,5,\dots\}$ for some $C, A \in \{3^2, 5^2, 7^2, \dots\}$ such that $p > q$ and $w > r$ and p, q, w, r, x are co-prime and $n \mid pq$ in view of $n \notin \{3^2, 5^2, 7^2, \dots\}$:

$$\begin{aligned}\Big\{\Big[&(2pq)^n = B^n = \left(C^{\frac{n}{2}}\right)^2 - \left(A^{\frac{n}{2}}\right)^2 \wedge C = (wr)^2 \wedge A = x^2 \wedge \frac{(2pq)^n + (x^2)^n}{2pq + x^2} = \frac{(2pq)^n + (x^2)^n}{(r^2)^n} = \frac{(w^2\,r^2)^n}{(r^2)^n}\\ &= (w^2)^n \wedge (r^n)^2 - x^2 = 2pq \;\wedge\; (2 \mid pq \equiv 0)\Big] \in \mathbf{0}\Big\}.\end{aligned}$$

The proof is incomplete because it does not include the case for $C \in \{6,10,14,\dots\}$ and it does not include the cases: $C, A \in \{3,5,7,\dots\} \setminus \{3^2, 5^2, 7^2, \dots\}$ and $(n \mid A \vee n \mid C)$.

This is the remark 2.

IV. Two Complete Proofs Of The Beal's Conjecture

Definition 1. $\boldsymbol{cpf}(A, B, C) = \boldsymbol{p}$, where $\boldsymbol{p} \geq 2$ is the common prime factor of given solution $[A, B, C]$ of the equation $A^x + B^y = C^z$.

Conjecture 1 (Beal Conjecture). For some $x, y, z \in \{3,4,5\dots\}$ and for some $A, B, C \in \{1,2,3,\dots\}$ such that A, B and C have the common prime factor $\boldsymbol{p} \geq 2$:

$$A^x + B^y = C^z.$$

Or – For all $x, y, z \in \{3,4,5\dots\}$ the equation

$$A^x + B^y = C^z$$

has no primitive solutions $[A, B, C]$ in $\{1,2,3,\dots\}$.

Proof. Suppose that for some $x, y, z \in \{3,4,5,\dots\}$ the equation

$$A^x + B^y = C^z$$

has primitive solutions $[A, B, C]$ in $\{1,2,3,\dots\}$.

Then only one number out of $[A, B, C]$ is even, which is obviously. Moreover on the strength of FLT we assume that – For all $\sigma \in \{1,2,3,\dots\}$: $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x, y, z)$.

If $y > x \geq z$ with $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x, y, z)$, then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^z C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow$$

$$[(\boldsymbol{p}^{x-z} A^x + \boldsymbol{p}^{y-z} B^y = C^z \vee A^x + \boldsymbol{p}^{y-z} B^y = C^z) \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow \boldsymbol{cpf}(\boldsymbol{p}, C^z) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

If $y \geq z > x$ with $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x, y, z)$, then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^z C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, C) = \boldsymbol{p}] \Rightarrow$$

$$[A^x + \boldsymbol{p}^{y-x} B^y = \boldsymbol{p}^{z-x} C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, C) = \boldsymbol{p}] \Rightarrow \boldsymbol{cpf}(\boldsymbol{p}, A^x) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

If $y = x > z$ with $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x, y, z)$, then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^z C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow$$

$$[\boldsymbol{p}^{x-z} A^x + \boldsymbol{p}^{y-z} B^y = C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow \boldsymbol{cpf}(\boldsymbol{p}, C^z) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

If $z > x \geq y$ with $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x, y, z)$, then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^z C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow$$

$$[(\boldsymbol{p}^{x-y} A^x + B^y = \boldsymbol{p}^{z-y} C^z \vee A^x + B^y = \boldsymbol{p}^{z-y} C^z) \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow \boldsymbol{cpf}(\boldsymbol{p}, B^y) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

If $z \geq y > x$ with $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x, y, z)$, then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^z C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, B) = \boldsymbol{p}] \Rightarrow$$

$$[A^x + \boldsymbol{p}^{y-x} B^y = \boldsymbol{p}^{z-x} C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, B) = \boldsymbol{p}] \Rightarrow \boldsymbol{cpf}(\boldsymbol{p}, A^x) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

If $z = x > y$ with $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x, y, z)$, then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^z C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow$$

$$[\boldsymbol{p}^{x-y} A^x + B^y = \boldsymbol{p}^{z-y} C^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Rightarrow \boldsymbol{cpf}(\boldsymbol{p}, B^y) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

For all $x \in \{3,4,5,\dots\}$ and for all $a, b \in \{1,2,3,\dots\}$ and for consecutive $k \in \{1,2,3,\dots\}$ such that $a > b$ and a, b are co-prime:

$$(a^x - b^x = c \mid \cdot\, c^{xk}) \Rightarrow (ac^k)^x - (bc^k)^x = c^{xk+1} = B^{xk+1} = C^x - A^x.$$

Thus A, B and C have $\boldsymbol{p} \geq 2$ because $\boldsymbol{p} \mid c$. The number of the solutions $[A, B, C]$ is infinite. ♠

Example 1. For all $x \in \{3,4,5,\dots\}$ and for all $a \in \{2,3,4,\dots\}$ and for $k = 2$ and $(b = 1)$:

$$[(a^x - 1)^{2x} + (a^x - 1)^{2x+1} = (a(a^x - 1)^2)^x \quad [12]\,] \wedge (a^x - 1)^{xk} = c^{xk}.$$

For all $x \in \{3,4,5,\dots\}$ and for all $a \in \{2,3,4,\dots\}$ and for consecutive $k \in \{1,2,3,\dots\}$:

$$(a^x - 1^x = c \mid \cdot\, c^{xk}) \Rightarrow (ac^k)^x - c^{xk} = c^{xk+1} = B^{xk+1} = C^x - A^{xk}.$$

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Thus A, B and C have $\boldsymbol{p} \geq 2$ because $\boldsymbol{p} \mid c$. The number of the solutions $[A, B, C]$ is infinite. ♠

Suppose that for some $x \in \{3,4,5, \dots\}$ and for some $k, A, B, C \in \{1,2,3, \dots\}$ such that A, B and C are co-prime:

$$A^{xk} + B^{xk+1} = C^{x}.$$

Then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^{xk} A^{xk} + \boldsymbol{p}^{xk+1} B^{xk+1} = \boldsymbol{p}^{x} C^{x} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow$$

$$[\boldsymbol{p}^{xk-x} A^{xk} + \boldsymbol{p}^{xk-x+1} B^{xk+1} = C^{x} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow \boldsymbol{cpf}(\boldsymbol{p}, C^{x}) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

This is the example 1.

Suppose that for some $x \in \{3,4,5, \dots\}$ and for some $k \in \{1,2,3, \dots\}$ the equation

$$A^{x} + B^{xk+1} = C^{x}$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \dots\}$.

Then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^{x} A^{x} + \boldsymbol{p}^{xk+1} B^{xk+1} = \boldsymbol{p}^{x} C^{x} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow$$

$$[A^{x} + \boldsymbol{p}^{xk-x+1} B^{xk+1} = C^{x} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow \boldsymbol{cpf}(\boldsymbol{p}, C^{x}) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

For some $z, x \in \{3,4,5, \dots\}$ and for some $a, b \in \{1,2,3, \dots\}$ and for consecutive $k \in \{1,2,3, \dots\}$:

$$(0 < a^{z} - b^{x} = c \mid \cdot c^{zxk}) \Longrightarrow$$

$$0 < (ac^{xk})^{z} - (bc^{zk})^{x} = c^{zxk+1} = B^{zxk+1} = C^{z} - A^{x}.$$

Thus A, B and C have $\boldsymbol{p} \geq 2$ because $\boldsymbol{p} \mid c$. The number of the solutions $[A, B, C]$ is infinite. ♠

Suppose that for some $x, z \in \{3,4,5, \dots\}$ and for some $k \in \{1,2,3, \dots\}$ the equation

$$A^{x} + B^{xzk+1} = C^{z}$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \dots\}$.

Then for some $\boldsymbol{p}$ with $x < z$ we get:

$$[\boldsymbol{p}^{x} A^{x} + \boldsymbol{p}^{xzk+1} B^{xzk+1} = \boldsymbol{p}^{z} C^{z} \wedge \boldsymbol{cpf}(\boldsymbol{p}, C) = \boldsymbol{p}] \Longrightarrow$$

$$[A^{x} + \boldsymbol{p}^{xzk-x+1} B^{xzk+1} = \boldsymbol{p}^{z-x} C^{z} \wedge \boldsymbol{cpf}(\boldsymbol{p}, C) = \boldsymbol{p}] \Longrightarrow \boldsymbol{cpf}(\boldsymbol{p}, A^{x}) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

Or – Then for some $\boldsymbol{p}$ with $x > z$ we get:

$$[\boldsymbol{p}^{x} A^{x} + \boldsymbol{p}^{xzk+1} B^{xzk+1} = \boldsymbol{p}^{z} C^{z} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow$$

$$[\boldsymbol{p}^{x-z} A^{x} + \boldsymbol{p}^{xzk-z+1} B^{xzk+1} = C^{z} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow \boldsymbol{cpf}(\boldsymbol{p}, C^{z}) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

For all $x \in \{3,4,5, \dots\}$ and for all $a, b \in \{1,2,3, \dots\}$ and for consecutive $k \in \{1,2,3, \dots\}$ such that a, b are co-prime:

$$(a^{x} + b^{x} = C \mid \cdot C^{xk}) \Longrightarrow (aC^{k})^{x} + (bC^{k})^{x} = C^{xk+1} = A^{x} + B^{x}.$$

Thus A, B and C have $\boldsymbol{p} \geq 2$ because $\boldsymbol{p} \mid C$. The number of the solutions $[A, B, C]$ is infinite. ♠

Suppose that for some $x \in \{3,4,5, \dots\}$ and for some $k \in \{1,2,3, \dots\}$ the equation

$$A^{x} + B^{x} = C^{xk+1}$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \dots\}$.

Then for some $\boldsymbol{p}$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^x B^x = \boldsymbol{p}^{xk+1} C^{xk+1} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow$$

$$[A^x + B^x = \boldsymbol{p}^{xk-x+1} C^{xk+1} \wedge \boldsymbol{cpf}(\boldsymbol{p}, A) = \boldsymbol{p}] \Longrightarrow \boldsymbol{cpf}(\boldsymbol{p}, B^x) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠

For all $x, y \in \{3,4,5, \dots\}$ and for all $a, b \in \{1,2,3, \dots\}$ and for consecutive $k \in \{1,2,3, \dots\}$:

$$(a^x + b^y = C \mid \cdot C^{xyk}) \Longrightarrow (aC^{yk})^x + (bC^{xk})^y = C^{xyk+1} = A^x + B^y.$$

Thus A, B and C have $\boldsymbol{p} \geq 2$ because $\boldsymbol{p} \mid C$. The number of the solutions $[A, B, C]$ is infinite. ♠

Without loss for below proof we can assume that $x < y$.

Suppose that for some $x, y \in \{3,4,5, \dots\}$ and for some $k \in \{1,2,3, \dots\}$ the equation

$$A^x + B^y = C^{xyk+1}$$

has primitive solutions $[A, B, C]$ in $\{1,2,3, \dots\}$.

Then for some $\boldsymbol{p}$ with $x < y$ we get:

$$[\boldsymbol{p}^x A^x + \boldsymbol{p}^y B^y = \boldsymbol{p}^{xyk+1} C^{xyk+1} \wedge \boldsymbol{cpf}(\boldsymbol{p}, B) = \boldsymbol{p}] \Longrightarrow$$

$$[A^x + \boldsymbol{p}^{y-x} B^y = \boldsymbol{p}^{xyk-x+1} C^{xyk+1} \wedge \boldsymbol{cpf}(\boldsymbol{p}, B) = \boldsymbol{p}] \Longrightarrow \boldsymbol{cpf}(\boldsymbol{p}, A^x) > 1,$$

which is inconsistent with A, B and C are co-prime. ♠ This are two complete proofs.

Remark 3. For the formalities we give the proof in some intergral cases because Beal's conjecture is the generalization of FLT – For some $x, y, z \in \{3,4,5, \dots\}$ and for some $A, B, C, a, b, c \in \{1,2,3, \dots\}$ such that $(x, y, z) \neq (3,3,3), (4,4,4), (5,5,5), \dots,$ and A, B and C are co-prime:

$$\begin{aligned}
\Big\{ &\Big[A^x + \left(B^{\frac{y}{x}}\right)^x = \left(C^{\frac{z}{x}}\right)^x \wedge \boldsymbol{gcd}(x,y) = \boldsymbol{gcd}(x,z) = 1 \wedge b^x = B \wedge c^x = C\Big] \\
&\vee \Big[\left(A^{\frac{x}{y}}\right)^y + B^y = \left(C^{\frac{z}{y}}\right)^y \wedge \boldsymbol{gcd}(x,y) = \boldsymbol{gcd}(y,z) = 1 \wedge a^y = A \wedge c^y = C\Big] \\
&\vee \Big[\left(A^{\frac{x}{z}}\right)^z + \left(B^{\frac{y}{z}}\right)^z = C^z \wedge \boldsymbol{gcd}(x,z) = \boldsymbol{gcd}(y,z) = 1 \wedge a^z = A \wedge b^z = B\Big] \\
&\vee \Big[A^x + \left(B^{\frac{y}{x}}\right)^x = \left(C^{\frac{z}{x}}\right)^x \wedge x \mid y \wedge \boldsymbol{gcd}(x,z) = \boldsymbol{gcd}(y,z) = 1 \wedge c^x = C\Big] \\
&\vee \Big[A^x + \left(B^{\frac{y}{x}}\right)^x = \left(C^{\frac{z}{x}}\right)^x \wedge x \mid z \wedge \boldsymbol{gcd}(x,y) = \boldsymbol{gcd}(y,z) = 1 \wedge b^x = B\Big] \\
&\vee \Big[\left(A^{\frac{x}{y}}\right)^y + B^y = \left(C^{\frac{z}{y}}\right)^y \wedge y \mid x \wedge \boldsymbol{gcd}(x,z) = \boldsymbol{gcd}(y,z) = 1 \wedge c^y = C \wedge \left(A^{\frac{x}{y}}\right)^y + B^y = (c^z)^y\Big] \\
&\vee \Big[\left(A^{\frac{x}{y}}\right)^y + B^y = \left(C^{\frac{z}{y}}\right)^y \wedge y \mid z \wedge \boldsymbol{gcd}(x,y) = \boldsymbol{gcd}(x,z) = 1 \wedge a^y = A \wedge (a^x)^y + B^y = \left(C^{\frac{z}{y}}\right)^y\Big] \\
&\vee \Big[\left(A^{\frac{x}{z}}\right)^z + \left(B^{\frac{y}{z}}\right)^z = C^z \wedge z \mid x \wedge \boldsymbol{gcd}(x,y) = \boldsymbol{gcd}(y,z) = 1 \wedge b^z = B \wedge \left(A^{\frac{x}{z}}\right)^z + (b^y)^z = C^z\Big] \\
&\vee \Big[\left(A^{\frac{x}{z}}\right)^z + \left(B^{\frac{y}{z}}\right)^z = C^z \wedge z \mid y \wedge \boldsymbol{gcd}(x,y) = \boldsymbol{gcd}(x,z) = 1 \wedge a^z = A \wedge (a^x)^z + \left(B^{\frac{y}{z}}\right)^z = C^z\Big]\Big\},
\end{aligned}$$

which is false because FLT is true. ♠

This is the remark 3.

Corollary 1. *For some* $x, y, z \in \{3,4,5 \dots\}$ *the equation*

$$A^x + B^y = C^z$$

has solutions $[A, B, C]$ *in* $\{1,2,3, \dots\}$ *such that* A, B *and* C *have the common prime factor* $\boldsymbol{p} \geq 2$.

This is the corollary 1.

Corollary 2. *For all* $x, y, z \in \{3,4,5 \dots\}$ *and for all* $A, B, C \in \{1,2,3, \dots\}$ *such that* A, B *and* C *are co-prime*:

$$A^x + B^y \neq C^z.$$

This is the corollary 2.

V. The Proof Of The Erdős-Straus Conjecture

Conjecture 2 (Erdős–Straus Conjecture). For all $n \in \{2,3,4, \dots\}$ and for some $a, b, c \in \{1,2,3, \dots\}$:

$$\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Proof. For all $n \in \{2,4,6, \dots\}$ and for some $a, b, c \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{n}{2} = a \ \wedge \ \frac{n+2}{2} = b \ \wedge \ \frac{n(n+2)}{4} = c\right]. \ \spadesuit$$

For all $n \in \{3,7,11, \dots\}$ and for some $a, c, b \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \ \wedge \ \frac{n+1}{2} = a = c \ \wedge \ \frac{n(n+1)}{4} = b\right]. \tag{1}$$

Thus for all $n \in \{3,9,15,21,27,33,39,45,51,57,63, \dots\}$ and for some $a, c, b \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \ \wedge \ \frac{2n}{3} = a = c \ \wedge \ n = b\right],$$

and for all $n \in \{7,21,35,49,63,77,91,105,119,133,147, \dots\}$ and for some $a, c, b \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \ \wedge \ \frac{4n}{7} = a = c \ \wedge \ 2n = b\right],$$

and for all $n \in \{11,33,55,77,99,121,143,165,187,209, \dots\}$ and for some $a, c, b \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{c} + \frac{1}{b} \ \wedge \ \frac{6n}{11} = a = c \ \wedge \ 3n = b\right], \dots.$$

For all $n \in \{5,13,21, \dots\}$ and for some $a, b, c \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{3n+1}{4} = a \ \wedge \ \frac{n(3n+1)}{4} = b \ \wedge \ \frac{3n+1}{8} = c\right]. \tag{2}$$

For all $n \in \{5,11,17, \dots\}$ and for some $a, b, c \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{n+1}{3} = a \ \wedge \ n = b \ \wedge \ \frac{n(n+1)}{3} = c\right]. \tag{3}$$

On the strength of [7] for all $n \in \{97,111,125, \dots\}$ and for some $a, b, c \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{2(n+1)}{7} = a \ \wedge \ 2n = b \ \wedge \ \frac{2n(n+1)}{7} = c\right]. \tag{4}$$

For all $n \in \{3,17,31, \dots\}$ and for some $c, x \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{2n} + \frac{1}{c} + \frac{1}{c+x} \ \wedge \ \frac{4n^2-1}{7} = x \ \wedge \ \frac{2n+1}{7} = c\right]. \tag{5}$$

On the strength of [7] for all $n \in \{13,33,53, \dots\}$ and for some $a, b, c \in \{1,2,3, \dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{3n+1}{10} = a \ \wedge \ \frac{n(3n+1)}{4} = b \ \wedge \ \frac{3n+1}{2} = c\right]. \tag{6}$$

The key of this proof is the following sum of the subsets, which should be written in columns to determine the missing odd prime numbers, namely

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$$\{5,9,13,\dots\} =$$

$$\{5,25,45,\dots\} \cup \{9,29,49,\dots\} \cup \{13,33,53,\dots\} \cup \{17,37,57,\dots\} \cup \{21,41,61,\dots\}.$$

From $(2),(3),(4),(5),$ and (6) we will have by analogy yes, as well from (1).

Therefore, and on the strength of the Theorem 1 and the Theorem 2 we get –

For all $n \in \{n\colon\ n = 337 + 120k \ \wedge\ k \in [0,1,2,\dots] \ \wedge\ only\ \ 1, n \mid n\}$.

Or – For all $n \in \{n\colon\ n = 1009 + 120k \ \wedge\ k \in [0,1,2,\dots] \ \wedge\ only\ \ 1, n \mid n\}$.

Or – For all $n \in \{n\colon\ n = 1201 + 120k \ \wedge\ k \in [0,1,2,\dots] \ \wedge\ only\ \ 1, n \mid n\}$.

Or – For some $m, x, c \in \{1,2,3,\dots\}$ and for some $d \in \{2,4,6,\dots\}$ such that $2nm > d$:

$$\left[\frac{4}{n} = \frac{1}{nm} + \frac{1}{x+c} + \frac{1}{c} \Longrightarrow (4m-1)c^2 + [(4m-1)x - 2nm]c - nmx = 0\right] \Longrightarrow$$

$$\left[\Delta = [(4m-1)x]^2 + (2nm)^2 = (n^2+m^2)^2 \ \wedge\ x = \frac{n^2-m^2}{4m-1} \ \wedge\ c = \frac{nm+m^2}{4m-1}\right] \vee$$

$$\left[\Delta = [(4m-1)x]^2 + (2nm)^2 = \left(\frac{(2nm)^2+d^2}{2d}\right)^2 \ \wedge\ x = \frac{\frac{(2nm)^2}{2d} - \frac{d}{2}}{4m-1} \ \wedge\ c = \frac{nm + \frac{d}{2}}{4m-1}\right].$$

This is the proof.

Example 2. For $n = 337$ and for $m = 18$ and for some $x, c \in \{1,2,3,\dots\}$:

$$\left[\frac{4m-1}{nm} = \frac{x+2c}{(x+c)c} \ \wedge\ 71c^2 + (71x - 12132)c - 6066x = 0\right] \Longrightarrow$$

$$\left[\Delta = (71x)^2 + 12132^2 = (n^2+m^2)^2 \ \wedge\ x = \frac{337^2 - 18^2}{71} \ \wedge\ c = \frac{6066+324}{71} = 90\right].$$

Hence for all $n \in \{337,1011,1685,\dots\}$ and for some $a, b, c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge\ \frac{1685n}{337} = (x+c) = a \ \wedge\ 18n = b \ \wedge\ \frac{90n}{337} = c\right].$$

This is the example 2.

Example 3. For $n = 1009$ and for $m = 3$ and for some $x, c \in \{1,2,3,\dots\}$:

$$\left[\frac{4m-1}{nm} = \frac{x+2c}{(x+c)c} \ \wedge\ 3c^2 + (3x - 6054)c - 3027x = 0\right] \Longrightarrow$$

$$\left[\Delta = (3x)^2 + 6054^2 = (n^2+m^2)^2 \ \wedge\ x = \frac{1009^2 - 3^2}{11} \ \wedge\ c = \frac{3027+9}{11} = 276\right].$$

Hence for all $n \in \{1009,3027,5045,\dots\}$ and for some $a, b, c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge\ \frac{92828n}{1009} = (x+c) = a \ \wedge\ 3n = b \ \wedge\ \frac{276n}{1009} = c\right].$$

This is the example 3.

Example 4. For $n = 1201$ and for $m = 8$ and for some $x, c \in \{1,2,3,\dots\}$:

$$\left[\frac{31}{8n} = \frac{x+2c}{(x+c)c} \ \wedge\ 31c^2 + (31x - 19216)c - 9608x = 0\right] \Longrightarrow$$

$$\left[\Delta = (31x)^2 + 19216^2 = (n^2+m^2)^2 \ \wedge\ x = \frac{1201^2 - 8^2}{31} \ \wedge\ c = \frac{nm+m^2}{31}\right].$$

Hence for all $n \in \{1201,3603,6005,\dots\}$ and for some $a,b,c \in \{1,2,3,\dots\}$:

$$\left[\frac{4}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ \wedge \ \frac{46839n}{1201} = a \ \wedge \ 8n = b \ \wedge \ \frac{312n}{1201} = c\right].$$

This is the example 4.

VI. THE PROOF OF THE JEŚMANOWICZ'S CONJECTURE

The Theorem 1 or the Theorem 2 gives independently all primitive solutions of Pythagoras Equation.

Corollary 3. *For all* $\sigma,u,v \in \{1,2,3,\dots\}$ *such that* $\boldsymbol{gcd}(u,v) = 1$ *and* $u - v \in \{1,3,5,\dots\}$:

$$\left\{(u^2 - v^2)^\sigma = \left[\frac{(u+v)^\sigma + (u-v)^\sigma}{2}\right]^2 - \left[\frac{(u+v)^\sigma - (u-v)^\sigma}{2}\right]^2 \wedge (u^2 - v^2)^{2+\sigma} + (2uv)^{2+\sigma} < (u^2+v^2)^{2+\sigma} \wedge (u^2-v^2)^2 + (2uv)^2 = (u^2+v^2)^2 \wedge u^2 - v^2 + 2uv > u^2 + v^2\right\} \Rightarrow$$

– *For* $\sigma = 2$ *and for all* $u,v \in \{1,2,3,\dots\}$ *such that* $\boldsymbol{gcd}(u,v) = 1$ *and* $u - v \in \{1,3,5,\dots\}$:

$$\left\{(u^2-v^2)^\sigma = (u^2-v^2)^2 \wedge \left[\frac{(u+v)^\sigma + (u-v)^\sigma}{2}\right]^2 = \left[\frac{(u+v)^2 + (u-v)^2}{2}\right]^2 = (u^2+v^2)^2 \wedge \left[\frac{(u+v)^\sigma - (u-v)^\sigma}{2}\right]^2 = \left[\frac{(u+v)^2 - (u-v)^2}{2}\right]^2 = (2uv)^2\right\} \Rightarrow (u^2-v^2, 2uv, u^2+v^2).$$

This is the corollary 3.

Corollary 4. *For all* $\boldsymbol{p} \in \{2,3,4,\dots\}$ *and for all* $u,v \in \{1,2,3,\dots\}$ *such that* $\boldsymbol{gcd}(u,v) = 1$ *and* $u - v \in \{1,3,5,\dots\}$:

$$[\boldsymbol{p}(u^2-v^2)]^2 + [\boldsymbol{p}(2uv)]^2 = [\boldsymbol{p}(u^2+v^2)]^2.$$

This is the corollary 4.

Remark 4.

$$3^2 + 4^2 = 5^2 \Rightarrow 6^2 + 8^2 = 10^2 \Rightarrow [(3,1) = (u\_2, v\_2) \wedge u\_2 - v\_2 \notin \{1,3,5,\dots\}].$$

$$3^2 + 4^2 = 5^2 \Rightarrow 12^2 + 16^2 = 20^2 \Rightarrow [(4,2) = (u\_4, v\_4) \wedge \boldsymbol{gcd}(u\_4, v\_4) = 2 > 1].$$

However

$$3^2 + 4^2 = 5^2 \Rightarrow 9^2 + 12^2 = 15^2 \Rightarrow (u\_3, v\_3) \in \emptyset.$$

Thus on the strength of the Corollary 3 we get

$$(u^2 - v^2, 2uv, u^2 + v^2) = (5^2 - 4^2, 2 \cdot 5 \cdot 4, 5^2 + 4^2) = (9, 40, 41).$$

The Theorem 1 or the Theorem 2 gives independently all solutions of the Pythagoras Equation. This is the remark 4.

Example 5. If $y \geq z > x$ with $(2+\sigma, 2+\sigma, 2+\sigma) \neq (x,y,z)$, then for some $\boldsymbol{p}$ we get:

$$\{[(u^2-v^2)\boldsymbol{p}]^x + (2uv\boldsymbol{p})^y = [(u^2+v^2)\boldsymbol{p}]^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, u^2+v^2) = \boldsymbol{p}\} \Rightarrow$$

$$\{(u^2-v^2)^x + \boldsymbol{p}^{y-x}(2uv)^y = \boldsymbol{p}^{z-x}(u^2+v^2)^z \wedge \boldsymbol{cpf}(\boldsymbol{p}, u^2+v^2) = \boldsymbol{p}\} \Rightarrow$$

$$\boldsymbol{cpf}(\boldsymbol{p}, (u^2-v^2)^x) > 1,$$

which is inconsistent with $u^2 - v^2, 2uv$ and $u^2 + v^2$ are co-prime. ♠ This is the example 5.

On the strength of the Corollary 2 and the Corollary 3 and the Fermat's Last Theorem we obtain –

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Corollary 5. *For all* $x, y, z \in \{3,4,5 \dots\}$ *and for all relatively prime* $u, v \in \{1,2,3, \dots\}$ *such that* $u - v \in \{1,3,5, \dots\}$:

$$(u^2 - v^2)^x + (2uv)^y \neq (u^2 + v^2)^z.$$

This is the corollary 5.

Conjecture 3 (Jeśmanowicz Conjecture). For all $x, y, z \in \{1,2,3, \dots\}$ and for all $u, v \in \{1,2,3, \dots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5, \dots\}$:

$$(u^2 - v^2)^x + (2uv)^y \neq (u^2 + v^2)^z.$$

Proof. Suppose that for some $x, y, z \in \{1,2,3, \dots\}$ and for some $u, v \in \{1,2,3, \dots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5, \dots\}$:

$$(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z.$$

On the strength of the Theorem 1 – For some $x, y, z \in \{1,2,3, \dots\}$ and for some $u, v \in \{1,2,3, \dots\}$ such that $(x, y, z) \neq (2,2,2)$ and $\boldsymbol{gcd}(u, v) = 1$ and $u - v \in \{1,3,5, \dots\}$:

$$[(u^2 - v^2)^x + (2uv)^y = (u^2 + v^2)^z \wedge u^2 - v^2 + 2uv > u^2 + v^2 \wedge (u^2 - v^2)^2 + (2uv)^2 = (u^2 + v^2)^2]$$
$$\Rightarrow [(u^2 - v^2)^x = (u^2 - v^2)^2 \wedge (2uv)^y = (2uv)^2 \wedge (u^2 + v^2)^z = (u^2 + v^2)^2] \Rightarrow (x, y, z) = (2,2,2),$$

which is inconsistent with $(x, y, z) \neq (2,2,2)$. ♠

From the above it follows that

$$[(u^2 - v^2)^1 + (2uv)^2 < (u^2 + v^2)^2 \wedge (u^2 - v^2)^2 + (2uv)^1 < (u^2 + v^2)^2] \wedge$$
$$[(u^2 - v^2)^1 + (2uv)^3 > (u^2 + v^2)^2 \wedge (u^2 - v^2)^3 + (2uv)^1 > (u^2 + v^2)^2] \wedge$$
$$[(u^2 - v^2)^3 + (2uv)^2 < (u^2 + v^2)^3 \wedge (u^2 - v^2)^2 + (2uv)^3 < (u^2 + v^2)^3].$$

Further on the strength of the Corollary 5 the Jeśmanowicz's Conjecture is true. This is the proof.

VII. Disproof The Abc Conjecture

Conjecture 4 (ABC Conjecture). For all $\epsilon > 0$ there exist only finitely many triples (a, b, c) of positive co prime integers, with $a + b = c > d^{1+\epsilon}$, where d denotes the product of the distinct prime factors of the product abc. [11]

Disproof. *For all* $n \in \{1,3,5, \dots\}$ *and for all relatively prime* $A, B \in \{1,2,3, \dots\}$ *and for some* $g \in \{1,2,3, \dots\}$ [3] *and for some* $\epsilon \in \{1,2,3, \dots\}$:

$$A^n + B^n = (A + B)\frac{A^n + B^n}{A + B} = (A + B)g = a + b = \epsilon = c < d^{1+\epsilon},$$

where d *denotes the product of the distinct prime factors of the product* abc. This is the disproof.

VIII. The Short Proof Of The Goldbach's Conjecture

Conjecture 5 (Goldbach Conjecture). For all $Z \in \{6,8,10, \dots\}$ and for some $X, Y \in \mathbb{P}$:

$$Z = X + Y.$$

Proof.

$$\{6,8,10, \dots\} = \{\mathbf{6}, 12, \mathbf{18}, 24, \mathbf{30}, 36, \mathbf{42}, 48, \mathbf{54}, 60, \mathbf{66}, 72, \mathbf{78}, 84, \mathbf{90}, \dots\} \cup$$
$$\{8, \mathbf{14}, 20, \mathbf{26}, 32, \mathbf{38}, 44, \mathbf{50}, 56, \mathbf{62}, 68, \mathbf{74}, 80, \mathbf{86}, 92, \mathbf{98}, 104, \mathbf{110}, \dots\} \cup$$
$$\{\mathbf{10}, 16, \mathbf{22}, 28, \mathbf{34}, 40, \mathbf{46}, 52, \mathbf{58}, 64, \mathbf{70}, 76, \mathbf{82}, 88, \mathbf{94}, 100, \mathbf{106}, 112, \dots\}.$$

Thus

$$[3] \cup [\mathbf{9}, \mathbf{15}, \mathbf{21}, \mathbf{27}, \mathbf{33}, \mathbf{39}, \mathbf{45}, \mathbf{51}, \mathbf{57}, \mathbf{63}, \dots] \cup$$
$$[7,13,19, \mathbf{25}, 31,37,43, \mathbf{49}, \mathbf{55}, 61,67,73,79, \mathbf{85}, \mathbf{91}, \dots] \cup$$
$$[5,11,17,23,29, \mathbf{35}, 41,47,53,59, \mathbf{65}, 71, \mathbf{77}, 83,89, \mathbf{95}, \dots] = [3,5,7, \dots].$$

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Hence

$$\{6\} = \{Z\colon\ Z = X + Y \land X = Y = 3\} \lor \{8\} = \{Z\colon\ Z = X + Y \land X = 3 \land Y = 5\} \lor$$
$$\{14,20,26,\ldots\} =$$
$$\{Z\colon\ Z = X + Y \land X \leq Y \land X, Y \in \mathbb{P} \land X, Y \in [7,13,19,\mathbf{25},31,\ldots]\} \lor$$
$$\{10,16,22,\ldots\} =$$
$$\{Z\colon\ Z = X + Y \land X \leq Y \land X, Y \in \mathbb{P} \land X, Y \in [5,11,17,23,29,\mathbf{35},\ldots]\} \lor$$
$$\{12,18,24,\ldots\} =$$
$$\{Z\colon\ Z = X + Y \land X < Y \land X, Y \in \mathbb{P} \land X \in [5,11,17,23,29,\mathbf{35},\ldots] \land Y \in [7,13,19,\mathbf{25},31,\ldots]\},$$

whence it implies that for all $Z \in \{6,8,10,\ldots\}$ and for some $X, Y \in \mathbb{P}\colon\ Z = X + Y$.

Therefore we obtain

$$\{\mathbf{18},24,\mathbf{30},36,\mathbf{42},48,\mathbf{54},60,\mathbf{66},\ldots\} =$$
$$\{z\colon\ z = x + y \land x \leq y \land x, y \in [\mathbf{9},\mathbf{15},\mathbf{21},\mathbf{27},\mathbf{33},\ldots]\} \lor$$
$$\{34,40,46,52,58,64,70,\ldots\} =$$
$$\{z\colon\ z = x + y \land x < y \land x \in [\mathbf{9},\mathbf{15},\mathbf{21},\ldots] \land y \in [\mathbf{25},31,37,\ldots] - \mathbb{P}\} \lor$$
$$\{44,50,56,62,68,74,80,\ldots\} =$$
$$\{z\colon\ z = x + y \land x < y \land x \in [\mathbf{9},\mathbf{15},\mathbf{21},\ldots] \land y \in [\mathbf{35},41,47,\ldots] - \mathbb{P}\}.$$

Thus for all $z \in \{18,20,22,\ldots\} - \{20,22,26,28,32,38\}$ and for some $x, y \in \{3,5,7,\ldots\} - \mathbb{P}\colon\ z = x + y$.

Finally we get (the key of this proof is the common prime factor equal 3) the short proof, namely:

$$\{18,24,30,36,42,48,54,60,66,\ldots\} =$$
$$\{(3Z)\colon\ (3Z) = (3X) + (3Y) \land 3X \leq 3Y \land X, Y \in \mathbb{P} \land (3X),(3Y) \in [9,15,21,27,33,\ldots]\} \Longrightarrow$$
$$\{6,8,10,12,14,16,18,20,22,\ldots\} = \{Z\colon\ Z = X + Y \land X \leq Y \land X, Y \in \mathbb{P}\}.$$

This is the short proof.

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