Adaptive Observer Backstepping Control for Industrial Robot Manipulators Using IMU

Tran Xuan Kien¹, Do Duc Hanh²

¹R&D Department, Military Institute of Science and Technology, Hanoi, Vietnam
²Control Engineering Department, Military Technical Academy, Hanoi, Vietnam

Abstract—In this paper, an adaptive observer backstepping control for robot manipulators in the presence of external disturbances and parametric uncertainties is developed. Only link angular positions are measured as outputs using an Inertial Measurement Unit (IMU) based on MEMS and Kalman Filter instead of encoders. A good performance of the designed control is achieved despite the presence of disturbances, parameter uncertainties, system nonlinearities and payload changes for a real-time system of a single-link flexible-joint manipulator.

Keywords—Adaptive Observer Backstepping, IMU, MEMS, Kalman Filter, Robot Manipulator Control.

I. INTRODUCTION

Backstepping is a systematic and recursive design methodology for nonlinear feedback control that breaks down the controller into steps and progressively stabilizes each subsystem (“step back” the feedback signals towards the control input) [1]. The backstepping method is used in numerous applications including robotic systems [4], [5]. Backstepping design provides a standard procedure to select Lyapunov function. Backstepping design procedure is based on the Lyapunov stability theory and applied for nonlinear systems with strict feedback form and adaptive to parameter uncertainties. A disadvantage of backstepping design procedure is that all states of the system must be measurable. For unmeasured states, an observer should be available to estimate. In this paper, an adaptive observer backstepping control is designed for nonlinear systems with parameter uncertainties, where only the output is measured and it is the only feedback for closed control loop, system nonlinearities are functions of the output. In the design procedure, the backstepping controller and observer are computed simultaneously. Update laws in the design procedure guarantee adaptability to parameter uncertainties.

The rest of the paper is structured as follows. In section II, an adaptive observer backstepping control design for a single-link flexible-joint robot manipulator is presented. In section III we briefly introduce an Inertial Measurement Unit [2] and how it is installed and used instead of encoders to measure angular positions of the robot arm.

II. ADAPTIVE OBSERVER BACKSTEPPING FOR SINGLE-LINK FLEXIBLE-JOINT ROBOT MANIPULATOR

A. System Modeling

We consider a single-link flexible-joint robot manipulator actuated by a DC motor (see Fig. 1).

![Fig. 1. A single-link flexible robot manipulator](image)

The dynamic equations of the system are as follows:

\[
J_1\ddot{q}_1 + F_1\dot{q}_1 + K\left(q_1 - \frac{q_2}{N}\right) + mgd\cos q_1 = 0
\]

\[
J_2\ddot{q}_2 + F_2\dot{q}_2 - \frac{K}{N}\left(q_1 - \frac{q_2}{N}\right) = K_i
\]

\[LD\dot{i} + Ri + K_i q_2 = u\]  \(1\)

Where \(q_1\), \(q_2\) are the angular positions of the link and the motor shaft, \(i\) is the armature current, and \(u\) is the armature voltage.
The inertias $J_1$, $J_2$, the viscous friction constants $F_1$, $F_2$, the spring constant $K$, the torque constant $K$, the torque constant $K$, the back-EMF constant $K$, the armature resistance $R$ and inductance $L$, the link mass $M$, the position of link’s centre of gravity $d$, the gear ratio $N$ and the acceleration of gravity $g$ can all be unknown.

B. The design procedure of adaptive observer backstepping control

We assume that only the link position $q_1$ is measured. The choice of state variables:

$$\zeta_1 = q_1, \zeta_2 = q_1, \zeta_3 = q_2, \zeta_4 = q_2, \zeta_5 = i$$

The dynamic equations of the system become:

$$\begin{align*}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= -\frac{mgd}{J_1}\cos y - \frac{F_1}{J_1}\zeta_2 - \frac{K}{J_1}\left(\zeta_1 - \frac{\zeta_2}{N}\right) \\
\dot{\zeta}_3 &= \zeta_4 \\
\dot{\zeta}_4 &= -\frac{K}{J_2N}\left(\zeta_1 - \frac{\zeta_2}{N}\right) - \frac{F_2}{J_2}\zeta_4 + \frac{K}{J_2}\zeta_5 \\
\dot{\zeta}_5 &= -\frac{R}{L}\zeta_5 - \frac{Kb}{L}\zeta_4 - \frac{1}{L}u \\
y &= \zeta_1
\end{align*}$$

(2)

Clearly, (2) is not in the output-feedback. Differentiating $y$ twice, we obtain $\ddot{\zeta}_2 = Dy$ ($D = \frac{d}{dt}$ is the differentiation operator) and

$$D'y = -\frac{mgd}{J_1}\cos y - \frac{F_1}{J_1}Dy - \frac{K}{J_1}\left(y - \frac{\zeta_2}{N}\right)$$

It implies that:

$$\begin{align*}
\zeta_3 &= \frac{JN}{K}\left(D'y + \frac{mgd}{J_1}\cos y + \frac{F_1}{J_1}Dy - \frac{K}{J_1}y \right) \\
\zeta_4 &= D\zeta_3 = \frac{JN}{K}\left(D'y + \frac{mgd}{J_1}D\cos y + \frac{F_1}{J_1}D^2y + \frac{K}{J_1}Dy \right)
\end{align*}$$

Differentiating and substituting $\zeta_3, \zeta_4$, we obtain

$$\begin{align*}
\zeta_3 &= \frac{JN}{K}\left(D'y + \frac{mgd}{J_1}D\cos y + \frac{F_1}{J_1}D^2y + \frac{K}{J_1}Dy \right) + \frac{mgd}{J_1}D'\cos y + \frac{F_1}{J_1}D^2\cos y + \frac{K}{J_1}D^3y \\
\dot{\zeta}_3 &= \frac{JN}{K}\left(D'y + \frac{mgd}{J_1}D\cos y + \frac{F_1}{J_1}D^2y + \frac{K}{J_1}Dy \right) + \frac{mgd}{J_1}D'\cos y + \frac{F_1}{J_1}D^2\cos y + \frac{K}{J_1}D^3y
\end{align*}$$

Finally, differentiating and substituting $\zeta_3, \zeta_4$ we arrive at the input-output description

$$\begin{align*}
D'y &= \frac{KK}{JJ,NNL}u - \left(R + \frac{F_1}{J_1} + \frac{F_2}{J_2}\right)D'y - \frac{mgd}{J_1}D'\cos y \\
+ \left[R\frac{K}{J_1} + \frac{KF_2}{J_2} + \frac{FK}{J_1N_1} + \frac{FK}{J_1J_2L} + \frac{FK}{J_1J_2L} + \frac{FK}{J_2L}\right]D'y
\end{align*}$$

(3)

It is tedious but straightforward using (3) to find a choice of state variables.

$$\begin{align*}
\dot{x}_1 &= x_1 + \theta_1 y \\
\dot{x}_2 &= x_2 + \theta_2 y + \theta_3 \cos y \\
\dot{x}_3 &= x_3 + \theta_4 y + \theta_5 \cos y \\
\dot{x}_4 &= x_4 + \theta_6 y + \theta_7 \cos y \\
y &= x_i
\end{align*}$$

Where the unknown parameters $\theta_1, \theta_2, \theta_3, \ldots, \theta_6$ are defined as:
State-observers are designed as follows:

\[
\dot{\hat{x}} = \xi_0 + \sum_{j=1}^{p} \theta_j \xi_j + \sum_{j=0}^{p} b_j \nu_j + \varepsilon
\]

\( p = 8; m = 0 \)

\( A_0 \) is chosen so that \( A_0 = A - kc^T \) is Hurwitz.

\[
A_0 = \begin{bmatrix}
-k_{i_1} & 1 & 0 & 0 & 0 \\
-k_{j_2} & 0 & 1 & 0 & 0 \\
-k_{k_3} & 0 & 0 & 1 & 0 \\
-k_{l_4} & 0 & 0 & 0 & 1 \\
-k_{m_5} & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\( \dot{\xi}_0 = A_0 \xi_0 + ky + \phi_0(y) \)

\( \dot{\xi}_j = A_0 \xi_j + \phi_j(y), \quad 1 \leq j \leq p \)

\( \nu_j = A_0 u_j + e_j \)

\[
\dot{\varepsilon} = \dot{\hat{x}} - \left( \xi_0 + \sum_{j=1}^{p} \theta_j \xi_j + \dot{\nu}_0 \right)
\]

\( = Ax + \phi_0(y) + \sum_{j=1}^{p} \theta_j \phi_j + bu - \xi_0 - \sum_{j=1}^{p} \theta_j \xi_j - b\dot{\nu}_0 \)

\( = Ax + \phi_0(y) + \sum_{j=1}^{p} \theta_j \phi_j + bu - A_0 \xi_0 - ky - \phi_0(y) - \sum_{j=1}^{p} \theta_j \phi_j(x) - b\nu_0 \)

\( = \sum_{j=1}^{p} \theta_j (A_0 \xi_j + \phi_j(y)) - b\nu_0 \)

As \( y = c^T x \Rightarrow Ax - ky = (A - ke^T) x = A_0 x \)

\( \dot{\varepsilon} = A_0 x - A_0 \xi_0 - \sum_{j=1}^{p} \theta_j A_0 \xi_j - b\nu_0 = \)

\( = A_0 \left( x - \xi_0 - \sum_{j=1}^{p} \theta_j \xi_j - b\nu_0 \right) = A_0 \varepsilon \)

It means that \( \varepsilon = x - \hat{x} \) is exponential and converges to zero since \( A_0 \) is Hurwitz. Hence, virtual estimation \( \hat{x} \) of \( x \) is:

\[
\dot{\hat{x}} = \xi_0 + \sum_{j=1}^{p} \theta_j \xi_j + b\nu_0
\]

We are now ready to design our adaptive output-feedback controller.

**Step 1:** The control objective is to track the reference signal \( y_r(t) \) with output \( y \), so the first error variable \( z_1 \) is tracking error:
As \( y = x \Rightarrow \dot{y} = \dot{x} = x + \sum_{j=1}^{p} \theta_j \varphi_{j,1}(y) \).

The derivative of \( z_i \) is:
\[
\dot{z}_i = \dot{y} - \dot{y}_r = x + \sum_{j=1}^{m} \theta_j \varphi_{j,1}(y) - \dot{y}_r
\]

Since \( x_2 \) is not measured, it cannot be our virtual control. We replace it with the sum of its “virtual estimate” and the corresponding error:
\[
x_2 = \xi_{0,2} + \sum_{j=1}^{p} \theta_j \xi_{j,2} + \sum_{j=0}^{m} b_j \nu_{j,2} + \epsilon
\]

Substituting, we obtain
\[
\dot{z}_i = \xi_{0,2} + \sum_{j=1}^{p} \theta_j \left[ \varphi_{j,1}(y) + \xi_{j,2} \right] + \sum_{j=0}^{m} b_j \nu_{j,2} + \epsilon - \dot{y}_r
\]

with \( m = 0 \), \( z_i \) becomes:
\[
\dot{z}_i = \xi_{0,2} + \sum_{j=1}^{p} \theta_j \left[ \varphi_{j,1}(y) + \xi_{j,2} \right] + v_{0,2} + \epsilon - \dot{y}_r \quad (8)
\]

The choice of virtual in (8) is \( v_{0,2} \) because (7) reveals that the control \( u \) appears in the 4th derivative of \( v_{0,2} \), sooner than for any of the other variables in (8). If \( v_{0,2} \) were the control and the parameters \( \theta_1, \theta_2, \theta_3, \ldots, \theta_8 \) were known, then our choice of control law would be
\[
v_{0,2} = -c_1 \dot{z}_i - d_1 \dot{z}_i - \xi_{0,2} - \dot{y}_r - \sum_{j=1}^{p} \theta_j \left[ \varphi_{j,1}(y) + \xi_{j,2} \right]
\]
\[
\dot{z}_i = -c_1 \dot{z}_i - d_1 \dot{z}_i - \xi_{0,2} + \epsilon
\]
\[
+ \left[ c_1 \dot{z}_i + d_1 \dot{z}_i + \xi_{0,2} - \dot{y}_r \right] + \sum_{j=1}^{p} \theta_j \left[ \varphi_{j,1}(y) + \xi_{j,2} \right] + v_{0,2}
\]
\[
\dot{z}_i = -c_1 \dot{z}_i - d_1 \dot{z}_i + \epsilon + \left( v_{0,2} + \theta_1 \omega_1 - \dot{y}_r \right)
\]

We introduce the second error variable as
\[
z_2 = v_{0,2} - \alpha_1 - \dot{y}_r
\]

And substitute it into (8) we obtain
\[
\dot{z}_i = -c_1 \dot{z}_i - d_1 \dot{z}_i + \epsilon + \theta_1^{T} \omega_1 + z_2 + \alpha_1
\]
\[
\theta_1^{T} = [1, \ \theta_1, \ \theta_2, \ldots, \ \theta_8, \ \alpha_1]
\]
\[
\alpha_1^{T} = [c_1 \dot{z}_i + d_1 \dot{z}_i + \xi_{0,2} + \varphi_{0,1}, \varphi_{1,1} + \xi_{2,1}, \varphi_{0,3}, \varphi_{1,3} + \xi_{3,1}, v_{0,2}]
\]

Denoting the first estimate of \( \theta_1 \) as \( \theta_1 \), the first stabilizing function \( \alpha_1 \) is chosen to be:
\[
\alpha_1 = -\theta_1^{T} \cdot \omega_1
\]

With this choice, the \( \dot{z}_1 \) equation becomes
\[
\dot{z}_1 = -c_1 \dot{z}_i + z_2 - d_1 \dot{z}_i + \left( \bar{\theta}_0 - \theta_1 \right)^{T} \cdot \omega_1 + \epsilon_2
\]

Our first Lyapunov function is
\[
V_1 = \frac{1}{2} \dot{z}_1^2 + \left( \bar{\theta}_0 - \theta_1 \right)^{T} \left( \bar{\theta}_0 - \theta_1 \right) + \frac{1}{d_1} \epsilon^{T} P_0 \epsilon
\]

where \( P_0 = P_0^{T} > 0 \), satisfies \( P_0 A_0 + A_0^{T} P_0 = -1 \). The derivative of \( V_1 \) is
\[
\dot{V}_1 = z_1 \dot{z}_1 - \left( \bar{\theta}_0 - \theta_1 \right)^{T} \cdot \dot{\theta}_1 + \frac{1}{d_1} \frac{d}{dt} (\epsilon P_0 \epsilon)
\]

\[
= z_1 \dot{z}_1 - c_1 \dot{z}_i - d_1 \dot{z}_i + \left( \bar{\theta}_0 - \theta_1 \right)^{T} \left( z_1 \omega_1 - \dot{\theta}_1 \right) + z_1 \epsilon_2 - \frac{1}{d_1} \epsilon^{T} \epsilon
\]

\[
= z_1 \dot{z}_1 - c_1 \dot{z}_i + \left( \bar{\theta}_0 - \theta_1 \right)^{T} \left( z_1 \omega_1 - \dot{\theta}_1 \right) - d_1 \left[ z_1 - \frac{1}{2d_1} \epsilon^T \epsilon \right] - \frac{3}{4d_1} \epsilon^2
\]

(10)

The \( (\bar{\theta}_0 - \theta_1) \)-term is now eliminated from (10) by the choice of update law
\[
\dot{\theta}_1 = \alpha_1 \dot{z}_1
\]

(11)

which yields
\[
\dot{V}_1 \leq z_1 \dot{z}_1 - c_1 \dot{z}_i - \frac{3}{4d_1} \epsilon^{T} \epsilon
\]

Step 2: The derivative of \( z_2 = v_{0,2} - \alpha_1 - \dot{y}_r \) is expressed as
\[
\dot{z}_2 = \dot{v}_{0,2} - \alpha_1 - \dot{y}_r
\]

\[
= -k_2 v_{0,2} + v_{0,3} - \frac{\partial \alpha_1}{\partial y} \left( \xi_{0,2} + \epsilon_2 \right) + \theta_1^{T} \omega_1 - \frac{\partial \alpha_1}{\partial y} \dot{y}_r
\]

(12)
$$- \frac{\partial \alpha_t}{\partial \xi_0} (A_0 \xi_0 + k, y + \varphi(y)) + \overline{\theta}^T \omega_t + \sum_{j=0}^m \frac{\partial \alpha_t}{\partial \xi_j} (A_0 \xi_j + \varphi(y))$$

$$+ \sum_{j=0}^m \frac{\partial \alpha_t}{\partial \xi_j} (A_0 \xi_j + \varphi(y)) - \frac{\partial \alpha_t}{\partial \xi_j} \omega_t \xi_j - \frac{\partial \alpha_t}{\partial \xi_j} \dot{y}_j - \dot{y}_j,$$

where

$$\dot{y}_j = \xi_{0,2} + \sum_{j=0}^n \theta_j [\varphi(j) + \xi_{j,2}] + u_{0,2} + \xi$$

$$\dot{y}_1 = \alpha_1 \xi_1$$

$$\overline{\theta}^T = [\theta_1, \theta_2, \theta_3, \ldots, \theta_s, 1]$$

$$\omega_2^T = - \frac{\partial \alpha_t}{\partial \xi_j} [\varphi_{1,1} + \xi_{1,2}, \ldots, \varphi_{k,1} + \xi_{k,2}, v_{0,2}]$$

Using the estimate of $\theta_2$, the stabilizing function $\alpha_2$ is defined:

$$\alpha_2 = -c_2 \xi_2 - z_1 - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2 + k_2 v_{0,1} + \frac{\partial \alpha_1}{\partial \xi_0} (\xi_{0,2} + \varphi_0(y))$$

$$- \overline{\theta}^T \omega_2 + \frac{\partial \alpha_1}{\partial \xi_2} (A_0 \xi_2 + k, y + \varphi(y)) + \sum_{j=0}^m \frac{\partial \alpha_1}{\partial \xi_j} (A_0 \xi_j + \varphi(y))$$

$$+ \sum_{j=0}^m \frac{\partial \alpha_1}{\partial \xi_j} (A_0 \xi_j + \varphi(y)) - \frac{\partial \alpha_1}{\partial \xi_j} \omega_t \xi_j + \frac{\partial \alpha_1}{\partial \xi_j} \dot{y}_j$$

And from definition $z_3 = v_{0,3} - \alpha_2 - \ddot{y}_r$, we rewrite this equation as

$$z_2 = -c_2 \xi_2 - z_1 - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2 + \frac{\partial \alpha_1}{\partial \xi_j} \xi_j + z_3 + (\overline{\theta} - \theta_2) \omega_2$$

The derivative of the nonnegative function $V_2$

$$V_2 = V_1 + \frac{1}{2} \xi_2^2 - 2 \left( \overline{\theta} - \theta_2 \right)^T \delta_2 + \frac{1}{2} \left( \delta_2 \right)^T \delta_2 \overline{P}_2 \delta_2$$

is computed as

$$\dot{V}_2 = \dot{V}_1 + z_2 \dot{z}_2 - \left( \theta - \theta_2 \right)^T \dot{\theta}_2 + \frac{1}{2} \left( \delta_2 \right)^T \delta_2 \overline{P}_2 \delta_2$$

$$\leq z_2 \dot{z}_2 - c_2 \xi_2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2 - z_2 \dot{z}_3 + z_2 \dot{z}_3$$

$$+ (\overline{\theta} - \theta_2)^T \left( \omega_t \xi_2 - \dot{\theta}_2 \right) - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2^2 + \frac{\partial \alpha_1}{\partial \xi_j} \left( \xi_0 \dot{z}_2 \right) - \frac{1}{d_2} \left( \omega_t \xi_2 + \xi_0 \dot{z}_2 \right) \dot{z}_2$$

$$= z_2 \dot{z}_2 - c_2 \xi_2^2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2^2 - z_2 \dot{z}_3 + z_2 \dot{z}_3$$

$$+ (\overline{\theta} - \theta_2)^T \left( \omega_t \xi_2 - \dot{\theta}_2 \right) - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2^2 + \frac{\partial \alpha_1}{\partial \xi_j} \left( \xi_0 \dot{z}_2 \right) - \frac{1}{d_2} \left( \omega_t \xi_2 + \xi_0 \dot{z}_2 \right) \dot{z}_2$$

The (13) term is then eliminated with the update law

$$\dot{\theta}_2 = \alpha_2 \xi_2$$

which yields

$$V_2 \leq -c_2 \xi_2^2 - c_2 \dot{z}_2^2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2^2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2^2$$

$$+ (\overline{\theta} - \theta_2)^T \left( \omega_t \xi_2 - \dot{\theta}_2 \right) - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2^2 + \frac{\partial \alpha_1}{\partial \xi_j} \left( \xi_0 \dot{z}_2 \right) - \frac{1}{d_2} \left( \omega_t \xi_2 + \xi_0 \dot{z}_2 \right) \dot{z}_2$$

$$\leq z_2 \dot{z}_2 - c_2 \xi_2^2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2^2 - z_2 \dot{z}_3 + z_2 \dot{z}_3$$

$$+ (\overline{\theta} - \theta_2)^T \left( \omega_t \xi_2 - \dot{\theta}_2 \right) - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2^2 + \frac{\partial \alpha_1}{\partial \xi_j} \left( \xi_0 \dot{z}_2 \right) - \frac{1}{d_2} \left( \omega_t \xi_2 + \xi_0 \dot{z}_2 \right) \dot{z}_2$$

$$\leq z_2 \dot{z}_2 - c_2 \xi_2^2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2^2 - z_2 \dot{z}_3 + z_2 \dot{z}_3$$

$$+ (\overline{\theta} - \theta_2)^T \left( \omega_t \xi_2 - \dot{\theta}_2 \right) - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2^2 + \frac{\partial \alpha_1}{\partial \xi_j} \left( \xi_0 \dot{z}_2 \right) - \frac{1}{d_2} \left( \omega_t \xi_2 + \xi_0 \dot{z}_2 \right) \dot{z}_2$$

$$\leq z_2 \dot{z}_2 - c_2 \xi_2^2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2^2 - z_2 \dot{z}_3 + z_2 \dot{z}_3$$

$$+ (\overline{\theta} - \theta_2)^T \left( \omega_t \xi_2 - \dot{\theta}_2 \right) - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2^2 + \frac{\partial \alpha_1}{\partial \xi_j} \left( \xi_0 \dot{z}_2 \right) - \frac{1}{d_2} \left( \omega_t \xi_2 + \xi_0 \dot{z}_2 \right) \dot{z}_2$$

$$\leq z_2 \dot{z}_2 - c_2 \xi_2^2 - \frac{3}{4d_1} \xi_2^T \xi - c_2 \dot{z}_2^2 - z_2 \dot{z}_3 + z_2 \dot{z}_3$$

$$+ (\overline{\theta} - \theta_2)^T \left( \omega_t \xi_2 - \dot{\theta}_2 \right) - d_2 \left( \frac{\partial \alpha_1}{\partial \xi_j} \right)^2 \xi_2^2 + \frac{\partial \alpha_1}{\partial \xi_j} \left( \xi_0 \dot{z}_2 \right) - \frac{1}{d_2} \left( \omega_t \xi_2 + \xi_0 \dot{z}_2 \right) \dot{z}_2$$

Denoting the first estimate $\theta$ as $\theta_1$, the third stabilizing function $\alpha_3$ is chosen to be

$$\alpha_3 = \frac{\partial \alpha_3}{\partial \xi_0} (A_0 \xi_0 + k, y + \varphi(y)) + \sum_{j=0}^m \frac{\partial \alpha_3}{\partial \xi_j} (A_0 \xi_j + \varphi(y)) + \sum_{j=0}^m \frac{\partial \alpha_3}{\partial \xi_j} (A_0 \xi_j + \varphi(y)) - \frac{\partial \alpha_3}{\partial \xi_j} \omega_t \xi_j + \frac{\partial \alpha_3}{\partial \xi_j} \dot{y}_j - \dot{y}_j,$$
\[ a_j = -c_j z_j - z_j - d_j \left( \frac{\partial a_j}{\partial y} \right)^2 z_j + k_j v_{0j} + \frac{\partial a_j}{\partial y} (\xi_{0j} + \varphi_{0j}(y)) \]

\[ -\theta_j \omega_j + \frac{\partial a_j}{\partial \xi_j} (A_j \xi_j + k_j y + \varphi_j(y)) + \sum_{i=0}^{m} \frac{\partial a_j}{\partial \xi_i} (A_j \xi_i + \varphi_i(y)) + \frac{\partial a_j}{\partial \omega_j} z_j + \frac{\partial a_j}{\partial y} y_j \]

And definition \( z_4 = v_{04} - \alpha_3 - y_r^{(4)} \) we rewrite this equation as

\[ \dot{z}_4 = -c_4 z_4 - z_4 - d_4 \left( \frac{\partial a_4}{\partial y} \right)^2 z_4 + k_4 v_{04} + \frac{\partial a_4}{\partial y} (\xi_{04} + \varphi_{04}(y)) \]

The derivative of the nonnegative function

\[ V_4 = V_4 + \frac{1}{2} z_4^2 + \frac{1}{2} (\bar{\theta} - \theta_j)^T \Gamma^{-1}(\bar{\theta} - \theta_j) + \frac{1}{d_3} \varepsilon^T P_3 \varepsilon \]

is computed and

\[ \dot{V}_4 \leq z_4 z_4 - c_4 z_4 - \frac{3}{4 d_3} \varepsilon^T \varepsilon - c_4 z_4^2 - c_4 z_4 - z_4 z_4 + z_4 z_4 \]

\[ + (\bar{\theta} - \theta_j)^T (\bar{\omega}_j - \theta_j) - d_4 \left[ \frac{\partial a_4}{\partial y} \right] z_4 \left[ \frac{1}{2 d_4} \right] + \frac{1}{d_4} \varepsilon^T \varepsilon - \frac{1}{d_4} \varepsilon^T \varepsilon \]

\[ = z_4 z_4 - c_4 z_4 - \frac{3}{4 d_4} \varepsilon^T \varepsilon - \frac{3}{4 d_4} \varepsilon^T \varepsilon - c_4 z_4^2 - c_4 z_4^2 \]

\[ + (\bar{\theta} - \theta_j)^T (\bar{\omega}_j - \theta_j) - d_4 \left[ \frac{\partial a_4}{\partial \omega_j} \right] z_4 \left[ \frac{1}{2 d_4} \right] + \frac{3}{4 d_4} \varepsilon^T \varepsilon + \frac{3}{4 d_4} \varepsilon^T \varepsilon \]

\[ \leq z_4 z_4 - c_4 z_4 - c_4 z_4 - c_4 z_4 - \frac{3}{4 d_4} \varepsilon^T \varepsilon + \frac{3}{4 d_4} \varepsilon^T \varepsilon + \frac{3}{4 d_4} \varepsilon^T \varepsilon \]

\[ + (\bar{\theta} - \theta_j)^T (\bar{\omega}_j - \theta_j) \]

The \((\bar{\theta} - \theta_j)\)-term is then eliminated with the update law

\[ \theta_j = \omega_j z_4 \]

which yields

\[ \dot{V}_4 \leq z_4 z_4 - c_4 z_4 - c_4 z_4 - c_4 z_4 - \frac{3}{4 d_4} \varepsilon^T \varepsilon + z_4 z_4 \]

**Step 4:** From definition \( z_4 = x_4 - y_r^{(3)} - \alpha_3 \) the derivative of \( z_4 \) is expressed as

\[ z_4 = \dot{v}_{04} - \dot{\alpha}_3 - y_r^{(4)} = -k_4 v_{04} + v_{05} - \frac{\partial a_4}{\partial y} (\xi_{04} + \varphi_4(y)) + \bar{\theta}^T \omega_4 \]

\[ -\frac{\partial a_4}{\partial \xi_4} (A_4 \xi_4 + k_4 y + \varphi_4(y)) + \sum_{i=0}^{m} \frac{\partial a_4}{\partial \xi_i} (A_4 \xi_i + \varphi_i(y)) + \frac{\partial a_4}{\partial \omega_4} z_4 + \frac{\partial a_4}{\partial y} y_4 \]

And definition \( z_5 = v_{05} - \alpha_4 - y_r^{(4)} \) we rewrite this equation as

\[ z_5 = \dot{v}_{05} - \dot{\alpha}_4 - y_r^{(4)} = -k_5 v_{05} + v_{06} - \frac{\partial a_4}{\partial y} (\xi_{05} + \omega_4(y)) + \bar{\theta}^T \omega_5 \]

\[ -\frac{\partial a_4}{\partial \xi_5} (A_5 \xi_5 + k_5 y + \omega_4(y)) + \sum_{i=0}^{m} \frac{\partial a_4}{\partial \xi_i} (A_5 \xi_i + \omega_4(y)) + \frac{\partial a_4}{\partial \omega_5} z_5 + \frac{\partial a_4}{\partial y} y_5 \]

Denoting the first estimate \( \theta \) as \( \theta_4 \) the 4th stabilizing function \( \alpha_4 \) is chosen to be

\[ \alpha_4 = -c_4 z_4 - z_4 - d_4 \left( \frac{\partial a_4}{\partial y} \right)^2 z_4 + k_4 v_{04} + \frac{\partial a_4}{\partial y} (\xi_{04} + \varphi_{04}(y)) \]

\[ -\theta_4 \omega_4 + \frac{\partial a_4}{\partial \xi_4} (A_4 \xi_4 + k_4 y + \varphi_4(y)) + \sum_{i=0}^{m} \frac{\partial a_4}{\partial \xi_i} (A_4 \xi_i + \varphi_i(y)) + \frac{\partial a_4}{\partial \omega_4} z_4 + \frac{\partial a_4}{\partial y} y_4 \]

And definition \( z_5 = v_{05} - \alpha_4 - y_r^{(4)} \) we rewrite this equation as

\[ z_5 = \dot{v}_{05} - \dot{\alpha}_4 - y_r^{(4)} = -k_5 v_{05} + v_{06} - \frac{\partial a_4}{\partial y} (\xi_{05} + \omega_4(y)) + \bar{\theta}^T \omega_5 \]

The derivative of the nonnegative function

\[ V_4 = V_4 + \frac{1}{2} z_4^2 + \frac{1}{2} (\bar{\theta} - \theta_j)^T \Gamma^{-1}(\bar{\theta} - \theta_j) + \frac{1}{d_4} \varepsilon^T P_4 \varepsilon \]

is computed and

\[ \dot{V}_4 \leq z_4 z_4 - c_4 z_4 - c_4 z_4 - c_4 z_4 - \frac{3}{4 d_4} \varepsilon^T \varepsilon - c_4 z_4^2 - c_4 z_4^2 \]

\[ -c_4 z_4^2 + z_4 z_4 + (\bar{\theta} - \theta_j)^T (\bar{\omega}_j - \theta_j) - d_4 \left[ \frac{\partial a_4}{\partial y} \right] z_4 \left[ \frac{1}{2 d_4} \right] + \frac{3}{4 d_4} \varepsilon^T \varepsilon + \frac{3}{4 d_4} \varepsilon^T \varepsilon \]

\[ = -c_4 z_4^2 - c_4 z_4^2 - c_4 z_4^2 + \frac{3}{4} \frac{1}{d_4} + \frac{1}{d_4} + \frac{1}{d_4} \varepsilon^T \varepsilon \]
\[
+ (\delta - \delta)\left(\omega_4 z_4 - \hat{\delta}_4\right) - d_4 \left[\frac{\partial \alpha}{\partial y} z_i - \frac{1}{2d_4} \varepsilon^T \varepsilon + \frac{1}{4d_4} \varepsilon^T \varepsilon \right]
\leq -c_i z_i^2 - c_i z_i^2 - c_i^2 z_i^2 - c_i^2 z_i^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4} \right) \varepsilon^T \varepsilon + (\delta - \delta)\left(\omega_4 z_4 - \hat{\delta}_4\right)
\]

The \((\delta - \delta)\)-term is then eliminated with the update law:

\[
\hat{\delta}_4 = \omega_4 z_4
\]  

which yields

\[
\dot{V}_4 \leq -c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4} \right) \varepsilon^T \varepsilon
\]

\[
= -k_i v_{i,0,1} + u - \frac{\partial \alpha}{\partial y} (\tilde{\varepsilon}_{0i} + \varepsilon) + \Theta^T \omega_3 - \frac{\partial \alpha}{\partial y} (A_i \tilde{\varepsilon}_i + \psi_i(y)) + \sum_{i=0}^{n} \frac{\partial \alpha}{\partial \xi_i} (A_i \tilde{\varepsilon}_i + \psi_i(y)) - \frac{\partial \alpha}{\partial \xi_i} \omega_i z_i - \frac{\partial \alpha}{\partial \xi_i} \omega_i^T y_i
\]

where

\[
\Theta^T = [\theta_1, \theta_2, \theta_3, \ldots \theta_n]
\]

\[
\omega_i^T = -\frac{\partial \alpha}{\partial y} \left[\theta_1, \theta_2, \ldots, \theta_n, 1\right]
\]

Denoting the first estimate \(\theta\) as \(\hat{\theta}_4\) the control is thus chosen as

\[
u = -c_i z_i - d_4 \left[\frac{\partial \alpha}{\partial y} z_i + k_i v_{i,0,1} + \frac{\partial \alpha}{\partial y} (\xi_i + \phi_0(y)) - \hat{\theta}_4\right]
\]

\[
-\frac{\partial \alpha}{\partial \xi_i} (A_i \tilde{\varepsilon}_i + \psi_i(y)) + \frac{\partial \alpha}{\partial \xi_i} \omega_i z_i + \frac{\partial \alpha}{\partial \xi_i} \omega_i^T y_i - \frac{\partial \alpha}{\partial \xi_i} \omega_i^T y_i
\]

\[
\sum_{i=0}^{n} \frac{\partial \alpha}{\partial \xi_i} (A_i \tilde{\varepsilon}_i + \psi_i(y)) + \frac{\partial \alpha}{\partial \xi_i} \omega_i z_i + \frac{\partial \alpha}{\partial \xi_i} \omega_i^T y_i - \frac{\partial \alpha}{\partial \xi_i} \omega_i^T y_i
\]

\[
The derivative of the nonnegative function
\]

\[
V_i = V_i + \frac{1}{2} \varepsilon^T \varepsilon + \frac{1}{2} (\delta - \delta)^T \Gamma^{-1} (\delta - \delta) + \frac{1}{d_4} \varepsilon^T P \varepsilon
\]

is computed and

\[
V_i \leq z_i \varepsilon_i - c_i \varepsilon_i^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4} \right) \varepsilon^T \varepsilon
\]

\[
= -c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4} \right) \varepsilon^T \varepsilon
\]

\[
\leq -c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4} \right) \varepsilon^T \varepsilon
\]

\[
= -c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4} \right) \varepsilon^T \varepsilon
\]

\[
\leq -c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - \frac{3}{4} \left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \frac{1}{d_4} \right) \varepsilon^T \varepsilon
\]

The \((\delta - \delta)\)-term is then eliminated with the update law:

\[
\hat{\delta}_4 = \omega_4 z_4
\]
which yields
\[ \dot{V}_5 \leq -c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - c_i z_i^2 - \frac{3}{4} \left( \frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} \right) \dot{\varepsilon}^T \varepsilon \]
is rendered nonpositive:
\[ \dot{V}_5 \leq - \sum_{i=1}^{5} \left( c_i z_i^2 + \frac{3}{4d_i} \dot{\varepsilon}^T \dot{\varepsilon} \right) \quad (18) \]

Stability and convergence. Due to the piecewise continuity of \( y''(t) \) and the smoothness of the nonlinearities (4), the solution of the closed-loop adaptive system exists. Let its maximum interval of existence be \([0,t_f)\). On this interval, the nonnegative function \( V_5 \) is nonincreasing because of (18). Thus \( z_1, z_2, \ldots, z_5, \theta_1, \theta_2, \ldots, \theta_6 \) are bounded on \([0,t_f)\) by constants depending only on the initial conditions of adaptive system. Moreover \( \dot{\varepsilon} = A_{\varepsilon} \varepsilon \) shows that \( \varepsilon \) is bounded. The boundedness of all other signals on \([0,t_f)\) is established too. Since \( z_i \) and \( y_i \) are bounded, it follows \( y \) is bounded.

III. USING AN IMU INSTEAD OF ENCODERS TO MEASURE ANGULAR POSITIONS

Development and implementation of an Inertial Measurement Unit (IMU) to estimate the attitude of objects in space is introduced in [2], [3]. Micro-Electro-Mechanical System (MEMS) sensors are used to measure angular velocity and acceleration, a magnetic sensor (magnetometer) is used to calibrate against orientation drift, a GPS signal receiver is integrated and an extended Kalman filter algorithm is applied for real-time signal processing. To measure angular positions of the robot arm, a small and low-cost IMU is used instead of absolute encoders for installation convenience but ensures the required accuracy. Attachment and installation of encoders in the shaft of the robot arm is not easy. The IMU is attached at the end of the robot arm. It can actually be placed anywhere in the robot arm and needs no link to the shaft. Due to space limitations, we skip the detailed IMU implementation in this paper.

IV. EXPERIMENT DESIGN

A. Experiment and simulation setup

A 2-DOF flexible-joint robot arm control is implemented based on algorithms shown in section II and using an IMU described in section III to measure angular positions. The control algorithms are modelled in Simulink, run in real-time under Real Time Window Target utility of Real Time Workshop, and interface through a PCI1711 card from Advantech with the real world robot arm (see Fig. 2). This model consisted of MATLAB/Simulink algorithm, PCI1711 card, DC motor with PWM driver and IMU. Torque transmission between the actuator (DC motor) and the plant (industrial robot arm) is of spring drive as a flexible-joint.

![Fig. 2. Model of control algorithms](image)

To investigate the proposed control algorithms, we designed an industrial robot arm model, as depicted in Fig. 3, where

1- PWM driver and power amplifier
2- Industrial robot arm
3- Inertial Measurement Unit
4- Industrial computer using MATLAB/Simulink, Real-time Workshop with DAQ card PCI1711.
Fig. 3 shows DC motor, gear and spring-based joint, where:
1- DC motor
2- Spring-based joint
3- Industrial robot arm’s gear.

B. Simulation Environment

Matlab/Simulink has been used to simulate the proposed algorithms. Then, the simulated algorithms are implemented in a real-time hardware using Real-time Workshop with DAQ cards.

The Simulation model is built using normal Simulink blocks. States, signals, and model parameters can be viewed, modified in run-time. That allows to find an optimal control parameter set, and controller types as well.

The experiment result is shown in Fig. 5, where line 1 depicts the setpoint (for robot arm angle) change from -25° to +25°. The other lines show changes of the robot arm angle output signals for different controllers, namely:
2- adaptive observer backstepping+PID without payload
3- adaptive observer backstepping + PID with payload
4- PID without payload
5- PID with payload.
In this paper, an adaptive observer backstepping control for robot manipulators is developed. Only link angular positions are measured as outputs using a low-cost IMU based on MEMS and Kalman filter instead of encoders for easy installation but no accuracy suffering. The stability and good performance is achieved despite the presence of disturbances, parameter uncertainties, system nonlinearities and payload changes. The result is shown for a real-time system of a single-link flexible-joint manipulator, but the proposed algorithms can be extended and applied to more complex industrial robot manipulators.

V. CONCLUSIONS

The result shows that control and adaptability laws (11) (13), (14), (15), (17) from the proposed adaptive observer backstepping algorithms in section II ensure the stability and adaptability despite parameter uncertainties in the system. Using adaptive observer backstepping control algorithm combined with PID, the robot arm’s angle closely tracks the setpoint with static error 1°, response time 0.8s, while static error and response time are 2° and 11s for PID control respectively. In the experiment, even though the payload is three times heavier than the arm robot, the performance of the adaptive observer backstepping controller is almost unchanged (with static error 1° and response time 0.8s) and that proves the adaptability of the proposed control algorithms.

REFERENCES