Adaptive Control Nonlinear System with Inaccurate Data on the Relative Degree in Complete Parametric Uncertainty

Siddikov I. H.¹, Atajonov. M. O.², Karimov Sh. S.²
¹Tashkent State University of Technology, Department of Information Technology in Control, University st. 2, 100095 Tashkent, Republic of Uzbekistan
²Andijan Engineering Institute, Department of Automation of Engineering Production, Boburshoh st. 5, 170019 Andijan, Republic of Uzbekistan

Abstract - The paper considers the problem of stabilizing the output of non-stationary objects with unknown parametric objects with inaccurately given the relative degree under the action of unknown disturbance, delay and unaccounted dynamics. We introduce a modification obtained in earlier publications of the control algorithm, held closed-loop system stability analysis and discusses the conditions of stability control systems for the given class.

Keywords – adaptive algorithm, nonlinear object, unknown indignation, condition of stability, control.

I. INTRODUCTION

The challenges to overcome the adaptive control are relevant and are applied in practice in cases where the parameters or state variables of the control object are difficult or impossible to measure and the unknown disturbance acts on the object or parameters are modified during operation [1, 2].

This article is devoted to the development of the adaptive control algorithm objects with a linear part and a nonlinear unit in the feedback with full parametric uncertainty and the availability of variables under the action of an unknown delay as limited in amplitude perturbations and non-parasitic accounted dynamics.

II. CONCLUSIONS

Let the control object possess a non-linear property and is described by the following equation:

\[
\begin{align*}
\dot{X}_1(t) &= A_{X} X_1(t) + d_x(t) \phi(y(t-h)) + b_x v(t) + e_x f(t) \\
y(t) &= c_1^T X_1(t) \\
\mu \dot{X}_2(t) &= F \chi(t) + q \eta(t), \\
v(t) &= l^T \chi(t),
\end{align*}
\]

(1)

With scalar input and scalar output, where \(X_1(t) \in \mathbb{R}^n\) – vector system state variables (1); \(X_2(t) \in \mathbb{R}^r\) – vector system state variables (2); \(y(t) \in \mathbb{R}\) – measured output variable of the object; the function \(y(t) \in \mathbb{R}\) – is not measured; \(u(t) \in \mathbb{R}\) – control signal; \(A_{X}, F, b_x, c_x, d_x, e_x, q\) and \(l\) – matrices and vectors corresponding to the dimension with unknown coefficients; equation (2) is asymptotically stable dynamics, which is not included in the synthesis control law; number \(\mu > 0\) – determines the speed of the system (2); \(f(t)\) – limited amplitude disturbance; \(d_x(t)\) – vector bounded variables; \(\phi(t) = \phi(y(t-h))\) – smooth nonlinear function satisfying the conditions of the sectoral constraints in the form of

\[
|\phi(t)| = |\phi(y(t-h))| \leq C|y(t-h)|.
\]

(3)

Where the numbers \(C > 0\) and \(h > 0\) are unknown.

Needed to ensure that the convergence of the output variable of the system (1), (2) in a given neighborhood of \(\delta_0\) the equilibrium position for a finite time [4]

\[
|y(t)| \leq \delta_0, \quad \forall t \geq t_r.
\]

(4)

The system (1) can be written as follows:

\[
\begin{align*}
\dot{X}_1(t) &= A_{X} X_1(t) + \sum_{i=1}^n D_i d_i(t) \phi(y(t-h)) + b_x v(t) + e_x f(t), \\
y(t) &= C_{X}^T X_1(t),
\end{align*}
\]

(5)

where

\[
D_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix},
\]

\[
D_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}, \quad d_i(t) - components vector of variables \begin{bmatrix} d_1(t) & d_2(t) & \cdots & d_n(t) \end{bmatrix}.
\]
Rewrite system (5), (2) in the form of an input-output
\[ a(p) \phi(t) = b(p) y(t) + \sum_{i=1}^{n} g_i(p) d_i(t) \phi(y(t-h)), \quad (6) \]
\[ d(p) v(t) = c(p) u(t), \quad (7) \]

Where \( p = d/dt \) – the operator of differentiation; output variable \( y(t) \) measured;
\[ b(p) = b_m p^m + b_{m-1} p^{m-1} + \ldots + b_1 p + b_0, \]
\[ a(p) = p^r + a_{m-1} p^{m-1} + \ldots + a_1 p + a_0, \]
\[ e(p) = e_k p^k + e_{k-1} p^{k-1} + \ldots + e_1 p + e_0, \]
\[ d(p) = d_r p^r + d_{r-1} p^{r-1} + \ldots + d_1 p + d_0, \]
\[ c(p) = d(0) \] – polynomials with unknown parameters; \( m \leq n - 1 \); known maximum relative degree of the transfer function \( b(p) / a(p) \), equals \( p_{\text{max}} \); polynomial \( b(p) \)
Hurwitz and factor \( b_m > 0; \) \( \infty > h > 0 \) – unknown delay, polynomials \( g_i(p) \) defined as \( g_i(p) = c_{i+1}^{T} (pI - A) \).

In the present case, only output variable is measured \( y(t) \), but its derivatives are not measured, which makes it difficult to build the controller.

Control laws are chosen as follows:
\[ u(t) = -(k + \gamma) \alpha(p) \xi_1(t), \quad (8) \]
\[ \begin{align*}
\xi_1(t) &= \sigma \xi_1(t), \\
\xi_2(t) &= \sigma \xi_2(t), \\
\vdots \\
\xi_{p-1}(t) &= \sigma( -k_1 \xi_1(t) - k_2 \xi_2(t) - \ldots - k_{p-1} \xi_{p-1}(t) + k_p y(t) ).
\end{align*} \]

Where the number of \( k \) \( \geq 0 \) and polynomial \( \alpha(p) \) degrees \( p - 1 \) are selected so that the transfer function
\[ H(p) = \frac{\alpha(p) b(p)}{a(p) + k \alpha(p) b(p)} \]
was strictly positive real, positive parameter \( \gamma \) it serves to compensate for nonlinearity of \( \phi(y(t-h)) \), number \( \sigma > k \), and the coefficients \( k_i \)
calculated from the requirements of the system asymptotic stability (9) zero-input \( y(t) \).

Substituting (8) into (7), and then equation (6) and providing a model in the form of an input-output state, we obtain:
\[ x = A x(t) + b(k + \gamma) e_i(t - y(t)) + \beta f(t) + \sum_{i \in I} \Phi_i \Psi \phi(y(t-h)), \quad (10) \]
\[ y(t) = c^T x(t), \quad (11) \]

Where \( e_i(t) = y_i(t) - \hat{y}(t) \) and \( \hat{y}(t) = \frac{d(p)}{d(p)} \xi_i(t) \), \( x \in R^n \) – vector state variable model (12); \( A, b, \beta, \chi, \) and \( c \) – the transition matrix of the model input-output model to the input state output.

We rewrite (9) and (2) in vector-matrix form:
\[ \hat{\xi}(t) = \sigma(\Gamma \xi(t) + dk_i y(t)), \quad \xi(t) = \hat{H} ^T \xi(t), \quad \hat{y}(t) = I^T z(t), \quad (13) \]
\[ \mu \xi(t) = F \hat{z}(t) + q \xi(t), \quad \hat{y}(t) = I^T z(t), \quad (14) \]

Where \( \xi \in R^{p-1} \), \( \zeta \in R^p \) – vectors of state variables models (14) and (15) respectively;

\[ H = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \cdots & -k_{p-1} \end{bmatrix} \] - Hurwitz into force calculation model coefficients \( k_i \) (9).

\[ d = [0 \ 0 \ 0 \ \cdots \ 1]^T, \quad \hat{h} = [1 \ 0 \ 0 \ \cdots \ 0]^T ; H, \quad q \] and \( l \) – the transition matrix of the model input-output model to the input-output state.

Consider the vectors of deviations
\[ \eta_1(t) = \mu y(t) - z(t), \quad (15) \]
\[ \eta_2(t) = \mu y(t) - \xi(t), \quad (16) \]

Differentiating equation (15) and (16) we obtain
\[ \dot{\eta}_1(t) = \mu y(t) + \mu^2 F \eta_1(t) + \mu^3 q \xi_2(t), \quad (17) \]
\[ \dot{\eta}_2(t) = \mu y(t) + \sigma \eta_2(t), \quad (18) \]
\[ e_1(t) = y(t) - \hat{y}(t), \quad (19) \]
\[ e_2(t) = y(t) - \xi(t), \quad (20) \]

Where it was considered that \( dk_1 = -\Gamma \hat{h} \) and
\[ -Fl = q. \]
Positive definite matrix $R = R^T$ and $N = N^T$ satisfy the Lyapunov equations:

$$F^T R + RF = -Q_2, \quad (21)$$
$$G^T N + NT = -Q_3 \quad (22)$$

Where $Q_2 = Q_2^T$ and $Q_3 = Q_3^T$ - positive definite matrix.

In this case, we restate the control law (8), by entering into it an additional factor:

$$u(t) = -\tilde{k} \alpha(p) \frac{1}{(T_p + 1)^2} \xi(t), \quad (23)$$

Where $\alpha = \rho_{\text{max}} - 1$ and $\tilde{k} = k + \gamma$. We will build the knob to the maximum specified relative degree.

If $\rho \leq \rho_{\text{max}}$ the following is obtained:

$$y(t) = \frac{b(p)}{a(p)(Tp + 1)^2} v(t) + \frac{e(p)(Tp + 1)^{\gamma}}{a(p)(Tp + 1)^{\gamma}} f(t) + \sum_{i} g_i(p) \theta_i(Tp + 1)^{\gamma} \alpha(t)$$

$$v(t) = \frac{d(p)}{c(p)(Tp + 1)^{\rho - 1}} u(t) \quad (25)$$

Where $\xi = \rho_{\text{max}} - \rho$, and equation (25) describes the dynamics unrecorded.

To achieve this goal it is necessary for a sufficiently small $\mu$ increase options of $\gamma$ and $\sigma$, as well as $\sigma > \gamma$. However, taking into account the availability of additional varied parameter $T$ there are additional conditions: $\tilde{k} < T^{-1} < \sigma$. With that said, we can offer the following tuning algorithm. Parameter $\tilde{k} = k + \gamma$ selected according to the following algorithm:

$$\tilde{k}(t) = \int_{t_0}^{t} \lambda(\tau)d\tau, \quad (26)$$

Where the function $\lambda(t)$ is selected as follows:

$$\lambda(t) = \{\lambda_0; |y(t)| > \delta_0; \lambda_0 > 0 \quad (27)$$

The parameter $T$ is set as follows:

$$T^{-1}(t) = T_0 \tilde{k}^{-2}(t), \quad T_0 > 0, \quad (28)$$

The parameter $\sigma$ calculated on the basis of the algorithm:

$$\sigma(t) = \sigma_0 \left[ T^{-1}(t) \right]^{\alpha}, \quad \sigma_0 > 0. \quad (29)$$

In this way, coefficient $\tilde{k}$ is adjusted linearly (26), (27) up until the variable $y(t)$ do not fall into some small areas and the parameters $T^{-1}$ and $\sigma$ configured quadratically (28) and a power law higher degree (29) respectively. In the case of tracking the problem of limited amplitude set point influence function $y(t)$ algorithm (27) is replaced by the tracking error $e(t) = g(t) - y(t)$.

An example of a control algorithm

Consider the following system:

$$y(t) = \frac{0.21p^2 + p + 1}{p^2 - p^2 + 3p + 3} v(t) + \frac{p + 1}{p^2 - p^2 + 3p + 3} f(t) \quad (30)$$

$$+ \frac{6\sin(10t)p + 1}{p^2 - p^2 + 3p + 3} \arctg(0.5y(t - 1))$$

$$v(t) = \frac{100}{p + 100} u(t),$$

$$f(t) = 6 + 6\sin(2t) + 10\sin(10t). \quad (31)$$

It is known that the maximum relative level of the system $\rho = 2$. Regulator type has been constructed (34) (9) relative to the maximum extent (fig. 1).
Fig. 1. The transients in a control system (30), (31), (23), (9) and the variable adaptive controller tuning parameters

Thus, it is clear that the presented control law ensures the convergence of the output variable in a given neighborhood $\delta_0 = 0.5$

REFERENCES


