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Order Filters: Towards their Statistical Adaptation in the Course of Processing Periodic Signals

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***Abstract*— The weighted order statistics (WOS) filters possess a number of merits as compared to other ones, e. g., linear filters. However, one of specific features of the order filters is their nonlinearity. The above circumstance causes the considerable difficulties of an analytical estimation of their behavior. At the same time, their response depends on a number of factors. Thus, it is possible to suppose that the quality of processing a signal by the WOS filter is a casual event. For this reason, the statistical trials method seems to be promising in selecting more qualitative WOS filters projects. For solving the corresponding task, a set of the standard WOS and a special class of the CoPh WOS filters (filters bank) is attracted under condition of a variation of work frequencies of the CoPh WOS filter and others its parameters. The prompting motive of our research is the needs in noisy geophysical data processing and analysis, in particular, data of vibro seismic investigations. We intend to propose some experience gained in this field of research.**

***Keywords*— Order statistics filters, processing of periodic signals, *s*tatistical trials.**

I. Introduction

Generally, vibro-seismic data are mostly records of harmonic or frequency-modulated (or sweep) signals under the condition of some digitization frequency. Most often, methods of such signals analysis and their processing are in the field of harmonic analysis. In the given paper, the weighted order statistics (WOS) filters are offered for the processing of frequency-modulated (FM) and harmonic signals corrupted by noise and for their analysis.

The WOS filters possess a number of advantages in comparison with other filters [1]. These merits are the following: 1) a remarkable ability of the impulse noise removal, 2) noise robustness, 3) preservation of steps for a signal in the form of a telegraphic sequence (with such specific features as the first derivative discontinuity). Thus, in [2] studied the behavior of the median filter diagonally applied across seismic traces for producing lateral smoothing and found that the median filters preserved steps as compared to the low-pass and the linear filters. At the same time, according to [1], the length of a median filter should surpass the length of a linear filter in one and half times for the equal decrease of signal noise.

On the other hand, the response of such filters tends to zero while the filter length (n) approaches an integer number of signal periods. This circumstance demands a special attention when processing periodic signals and solving the problem of the corresponding adaptation of the WOS filters.

Among the papers dealing with the solution of the above problem, we will note [3, 4, 5], as they exploit different approaches. Here, the approach presented in [5] will used, where zero weights are added to an appropriate set while in [4] is used the hybrid (the WOS and the linear finite impulse response – FIR) filters, and in [3] admitted negative weights in the median filter performance.

Owing to nonlinearity of the WOS filters, the analytical estimation of their behavior is a very complicated process because it appreciably depends, on the one hand, on the filter project (values of scales, the size of the aperture or a window of the analysis, a sequence of operations in the multistage process of filtration) and, on the other hand, it depends on the form of a signal and specificity of noise. So, the results of the research in [2] show the importance of the filter design for attaining high attenuation levels of noise without causing a considerable signal distortion. Thus, it can be supposed that the response of the WOS filters is a casual event in the general case. Under these conditions, it is of interest the method of statistical trials to select the most efficient project of the WOS filter.

Thus, the task of interest is the processing of periodic signals and monitoring of the quality of the restored signals depending on using different projects of filters and their parameters. The basic points of this approach were discussed at the EAGE Conference [6]. At the same time, a specialized computer system [7] was utilized in the course of the above trials.

II. The Basic Definitions Of The WOS Filters

At first, it is worthwhile to give the basic definitions of such filters.

Let a periodic signal be one-dimensional time series $X=\{x_1,\dots, x_n\}$ recorded at discrete instants of time $t_1,\dots, t_n$ ($t_{i-1}-t_i=\Delta t=$const; $i=2,\dots, N$). Let a sequence

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$$Y=\{y_i: i=1,\ldots, n\} \qquad (1)$$

presents some samples of a signal which includes n quantities of numerical data.

The term "order filters" comes from the notion "a variational row" of mathematical statistics, where numerical values of a data row are arranged in increasing (or decreasing) order: $\tilde{y}_i$, $i=1,\ldots, n$. Here, the r^{th} order statistics $y_{(r,\ n)}$ is defined as the r^{th} quantity in size. Generally, it is possible to set the locus of any term $\tilde{y}_r$, $n \geq r \leq 1$ of a variational row by means of a relative number α of preceding elements (smaller or equal in size) and a relative number β of subsequent ones (equal to or greater than it): $\alpha=(r-1)/(n-1)$, $\beta=1-\alpha$. In this case, we have the form of the percentile filter. It is denoted as $y_{(\alpha,\ n)}$ if there exists a number $n_\alpha=(n-1)\cdot\alpha$ of $y_{i(\alpha)}$ quantities, and a number $n_\beta=(n-1)\cdot(1-\alpha)=(n-1)\cdot\beta$ of $y_{i(\beta)}$ quantities provided:

$$\left.\begin{array}{l} y_{i(\alpha)} \leq y_{(\alpha,n)}, \\ y_{i(\beta)} \geq y_{(\alpha,n)}, \\ y_{(\alpha,n)} \bigcup\limits_{1}^{n_\alpha} y_{i(\alpha)} \bigcup\limits_{1}^{n_\beta} y_{i(\beta)} = Y, \end{array}\right\} \qquad (2)$$

and $\alpha+\beta=1$.

Now, the formal definition of the order filtration procedure can be presented as a sequence of the following operations:

i) $Y=\{y_i: i=0, \pm1,\ldots, \pm v\}$ is the sampling of $n=2v+1$ signal values, where $Y \subset X$ (n is odd),

ii) $\tilde{Y} = \tilde{y}_{c-v} \leq \ldots \tilde{y}_c \leq \ldots \tilde{y}_{c+v}$ is the construction of a variational row, where the term $\tilde{y}_i$ is statistics of a corresponding order, $i=1,\ldots, n$.

iii) RANK($y_1,\ldots, y_{(n-1)/2},\ldots, y_n$) is the operation of replacement of the central term $y_v \in Y$ by the statistics $\tilde{y}_r \in \tilde{Y}$ ($v, c, r \in Z$, $1 \geq r \leq n$).

The last form is the definition of a standard order statistic filter. In a special case, if $r=(n-1)/2$ (i.e., $\alpha=0.5$), a filter is a median one: $\mathrm{MED}_n(y_1,\ldots,y_{(n-1)/2},\ldots,y_n) = \tilde{y}_c$.

Let us have a set W of the quantities w_i ($i=1,\ldots, n$), where each w_i is associated with a sample $y_i \in Y$. This $w_i \in W$ is called a weight and can be treated as a number of copies of the corresponding sample $y_i \in Y$. Weights are introduced for emphasizing some elements of a sequence [1, 8, 9]. The extended sequence Y thereby gains a new quality as a set with a number of elements $N = \sum_{i=1}^{n} w_i$.

At the same time, N is also odd. In this case, the form $\tilde{y}^{W}_{\alpha,N(w_0,w_1,\ldots,w_R)} = \mathrm{RANK}_{N(W)}(w_1 \times y_1,\ldots, w_{(n-1)/2} \times y_{(n-1)/2},\ldots, w_n \times y_n)$ is the definition of the WOS filter in the general case. This generalization allows a filter to keep properties of the median one, but changes the result of the filtering, i.e., $u_{0.5} \neq u^{(w)}_{0.5}$ [9].

Passing to the question of processing the periodic signals by the WOS filters, we can equalize to zero the weights of all the terms of the filter input sequence except the weight of the central element (CE) and the weights of such terms which are apart from one another by the length of a period [5]. Thus, the offered approach demands the knowledge only of the frequency band of a signal under processing unlike approaches proposed in [3, 4]. This allows us to project the so-called co-phased WOS (CoPh WOS) filters. At the same length equalizing to m periods of a signal (where time, an important condition of such a filter is providing the filter m is even (m=2,4, etc.). The above proposition is illustrated in Figure 1.

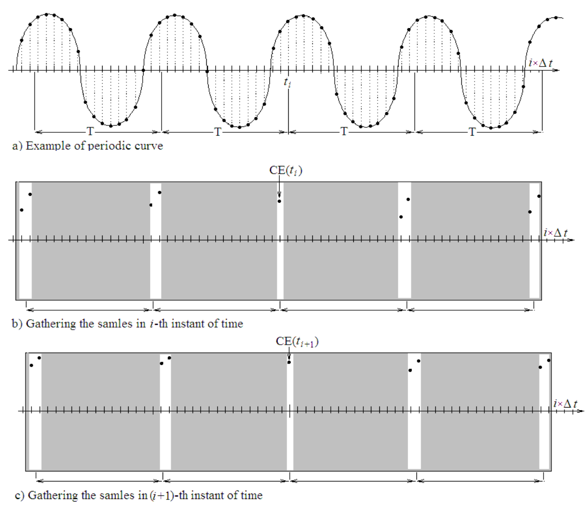

Figure 1. Sampling of periodic signal values closest to the CE phase at different instants of time.

According to the concept of weights, the corresponding set W of the CoPhWOS filter includes such a subset $W^* \subset W$ of weights w^*, where $\forall w^* \in W^*$: $w^* = 0$.

The procedure for calculating the weights of the CoPhWOS filter is as follows [5]:

1. Computation of the length of the filter as $[\hat{n}]$, where $n=2RT/\Delta t+1$;
2. Realization of assignments: $w_0=1$, $w_{\pm i}=0$, $i=1,\ldots, v$
3. Introduction of the integer variable $j=1,\ldots,R$ as a cycle index;

4. Computation of indices of the elements with nonzero weights, and computation of the corresponding weights according to the rules:

$$\left.\begin{array}{l} \text{i) } V_j = j\mathrm{T}/\Delta t; \\ \text{ii) if } V_j = [\widehat{V}_j] = [\breve{V}_j]\text{, then } w_{\pm V_j} = 1\text{; else:} \\ \text{iii) } w_{\pm j}(\pm[\widehat{V}_j]) = V_j - [\breve{V}_j]; \\ \text{iv) } w_{\pm j}(\pm[\breve{V}_j]) = [\widehat{V}_j] - V_j. \end{array}\right\} \quad (3)$$

5. If $j<R$, then the increment $j=j+1$, and go to item 4i; otherwise exit this loop and end the procedure.

Hire, T is the period of a corresponding frequency, $V_j=j\mathrm{T}/\Delta t$, and w_0 is the weight of the CE. At the same time, the symbol $[\,\widehat{x}\,]$ denotes the smallest integer (that is greater or equal to x or the nearest from above) and $[\,\breve{x}\,]$ is the largest integer, which is less or equal to x (the nearest from below). The frequency which defines values of the filter weights will be called the work frequency of a filter. Thus, a running order filter will test all the data of the source numerical sequence.

Let the terms $w_j([\,\widehat{v}_j\,])$ of procedure (2) be designated as $w_j\phi_j$. As defined in procedure (2), the index j of the corresponding term is nothing but the periods quantity between the CE and this filter term under the condition of symmetry of the node terms concerning the CE. Here, $j=0$, $1,\ldots,$ R and $\pm V_j=j\mathrm{T}/\Delta t$. Let us use the notation CoPhWOS$_R^{f,W}$ to designate such a filter, where R is a radius of the filter and f is the work frequency of the filter. Now, the CoPhWOS$_R^{f,W}$ filter of radius R is

$$\tilde{y}^{\,f,W}_{\alpha,N(k_0(\phi_0);k_{\pm1}(\pm\phi_1),\ldots,k_{\pm R}(\pm\phi_R))} =$$

$$\mathrm{MED}_{N(W)}(w_0\cdot y_0(\phi_0),\ w_{\pm1}\cdot y_1(\phi_{\pm1}),\ldots,\ w_{\pm R}\cdot y_{\pm R}(\phi_{\pm RV}))$$

where $w_0 \neq 0$.

Similarly, the definition of the standard WOS filter will be used in the form $\tilde{y}^{W}_{\alpha,N(k_0,k_1,\ldots,k_R)}$. Here, some terms can also include zero weights, however values of the corresponding filter data do not depend on a signal frequency.

The above algorithm is readily adapted to the case where the weight of the central element exceeds the weights of other components not less than by unit – for emphasizing its value. The algorithm in this case ensures the symmetry of weights as well. However, here, the weight of w_0(CE) should be odd. The non-integer weights can be transformed to integers with some loss of accuracy, by introduction an appropriate factor. Schemes for treatment with non-integer weights were offered in [10].

III. Specific Features Of The Method Of Statistical Trials For Searching The WOS Filters Projects

It is possible to note the following parameters of the WOS and the CoPhWOS filters which influence the quality of signal processing:

– values of the weights $W=\{w_1, w_2, \ldots\}$ of the WOS filters as well as the distribution of zero weights which are the functions of frequencies in the case of the CoPh WOS filters;

the size of the aperture or the filter window analysis;

a sequence of operations in the multistage process of filtration;

the type of operations;

percentiles of the WOS and the CoPh WOS filters;

swinging the work frequency;

the mode of counting the effects of signal digitization [11].

Some explanations will be given for the last two parameters which influence the quality of processing the periodic signals.

It is possible processing a periodic signal using two or three close enough work frequencies and subsequent application of averaging or the median filter diagonally (as, for example, in [2]) across the results of the processing.

As for the effects of signal digitization, they can be explained by using Figure 2.

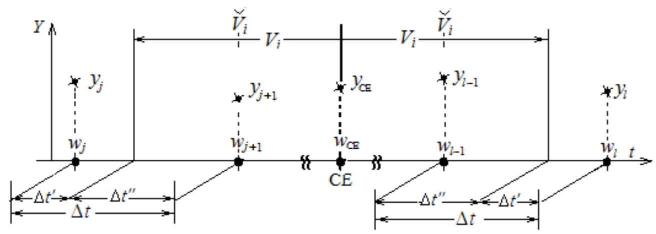

Figure 2. An example of digitization of a periodic signal and weights distribution of the CoPh WOS filter.

The signal data and corresponding weights of filter units can be processed as they arrive at a filter input, or two neighboring units can be pooled in a certain unit. We propose to attract the pooling in the form:

$$y_j+(y_{j+1} - y_j)\times\Delta t_j',\ w_j= w_j + w_{j+1}.$$

In the general case, a filter as the whole can include some ordered set (a sequence) of separate filters of signal processing. Let such singled filter be called a filter node. At the same time, such node includes some "filter terms" sequence of length n. Further, we will note that using only single the WOS or the CoPh WOS filter for signal processing can be considered as a separate case.

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Generally, the sequence of operations of an order filtration is involved for signal processing. Such sequence can be understood as adjoint oriented graph of processing where this or that single the order filter is compared for separate node. Basically, attraction of some set of filters (filters bank, or graphs bank of processing) is of interest for signals processing.. The above proposition allows us to speak about one more possibility of variational parameter, i.e., changing the used processing graph by the other.

The CoPhWOS filter satisfies the requirement of discarding those samples in the window which are not in phase with the center of the window [4] if a signal is a harmonic one on the whole length of its existence. However, in the course of processing the FM signal, the above condition is not satisfied. According to the research [5], such a filter of the two-period length of a FM signal will save the width Δf(CoPhWOS)∈1.0÷1.5 Hz in its frequency band. That is, if Δf(FM) > Δf(CoPhWOS), it is needed to attract a special technique for restoration of a source signal in the whole frequency band. Such a case was studied by [12], however, it is not considered here.

IV. Some Results Of Signal Processing

The methodology in question was investigated on the models of linear frequency-modulated signals using numerical modeling. Parameters and characteristics of the corresponding signals are the following: the start time of a sounding signal is 0 s; the arrival time of the sounding signal in a noise signal is 4 s (i.e., the sounding signal is recorded when removing from a vibration source). At the same time, the bandwidth of the sounding signal is 7.2 Hz ÷8.2 Hz, the digitization frequency being Δt=0.08 s. as for a sounding signal and for a noised signal. The white noise with zero average Gaussian distribution was used for obtaining a noise signal model. At the same time, the period of the signals recording is 1100 sec.

The results of the experiments obtained under condition of three cases of noise levels are shown in Table 1, where $\check{s}/\xi$ is the signal-to-noise ratio obtained by means of convolution of a noise signal with a sounding signal. At the same time, the estimations of the ratio of the mean square deviation of the sounding signal and the model of noise signal data before the processing is s/n.

TABLE I
RESULTS OF CONVOLUTION OF A FILTERED NOISE SIGNAL WITH THE SOUNDING SIGNAL

Before Processing					
Case 1 $\check{s}/\xi$=99.4; s/n=0.2		Case 2 $\check{s}/\xi$=87.6; s/n=0.1		Case 3 $\check{s}/\xi$=10.1; s/n=0.01	
After Processing					
***f* Hz**	$\check{s}/\xi$	***f* Hz**	$\check{s}/\xi$	***f* Hz**	$\check{s}/\xi$
7.75	125.6	7.7	93.0	7.65	14.4
7.765	127.0	7.71	96.8	7.67	15.9
7.77	147.5	7.72	94.1	7.68	10.2
7.775	137.8	7.73	92.4	7.69	10.1
7.785	145.7	7.74	91.0		

The results presented in the above Table are obtained by means of the filter structure presented by corresponding graph in Figure 3.

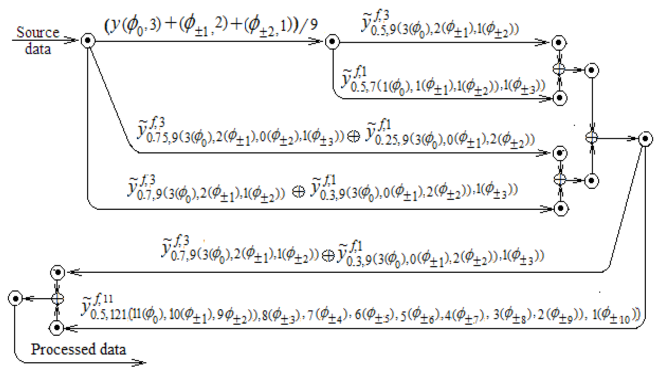

Figure 3. The used order filter structure where ⊙ denote a graph node (a point of data receiving), graph edges present contents of appropriate single filters, and ⊕ denote the operation of composition.

Figure 4 presents, from above downward, the model of a noised signal (the result of composition of a sounding signal and additive white noise with the value of s/n=0.2), the results of processing, and the sounding signal.

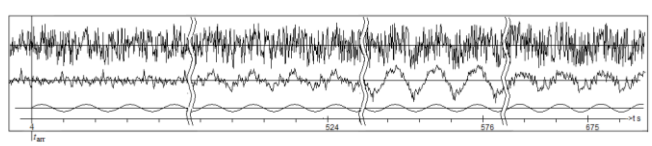

Figure 4. The image (from above downward) of the model of the noise signal, the noise signal after the processing, and the sounding signal, where t_{arr} is the arrival time of the sounding signal in the recorded signal.

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Finally, we will present the results of the experiments obtained by using the other filter structure. The corresponding flow-chart is depicted in Figure 5, and obtained data are shown in Table II.

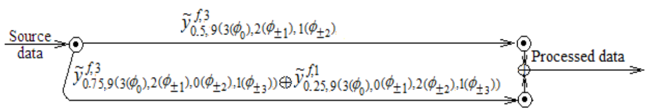

Figure 5. The adjoint oriented graph of processing used in the second experiments (the denotes are analogues of the previous case).

TABLE II
RESULTS OF CONVOLUTION OF A FILTERED NOISE SIGNAL WITH THE SOUNDING SIGNAL

Before Processing	
$\check{\xi}/\xi$ = 13.374, s/n = 0.066	
After Processing	
***f* Hz**	$\check{\xi}/\xi$
7.975	11.402
8.000	12.855
8.025	12.463
8.050	14.281
8.075	14.493
8.100	13.513
8.125	12.418

V. CONCLUSION

In this paper, the approach to the statistical adaptation of the WOS filters for processing the frequency-modulated signals is proposed. The basic points of this approach are: attracting a filters bank, a variation of the filters data, and parameters of the processing. The corresponding algorithm of filtering the FM signals under the conditions of statistical trials of the WOS filters was demonstrated using the noise FM signal. The results of the experiments conducted demonstrate the dynamics of a considerable dependence of the signals quality processing from the value of the work frequency of the CoPhWOS filter and its structure including values of the corresponding data. It can be supposed that the results obtained demonstrate the possibility of the approach proposed which is rather effective even in the case of very small the signal-to-noise ratio.

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