Abstract - Estimating the target path with Radar and Sonar, when it is in a motion is a nonlinear state estimation problem. The target parameters are measured with sensors. This sensor gives us the polar coordinates values like range, range rate, and about two angles. To avoid nonlinear filters, the polar coordinate values are converted to Cartesian coordinates gives better performance in estimating the target path. This conversion of polar to Cartesian is referred as converted measurement Kalman filtering (CMKF). Here two contributions are taken for conversion of parameters. One is converted measurement Kalman filter (CDMKF) for exploiting range rate measurements and other is converted position measurement Kalman filter (CPMKF). Taking these two methods parallel tends to new state estimator called Fused Kalman filter (FKF). The resulting states of CDMKF and CPMKF are combined by static minimum mean squared error estimator results final state estimates. This work refers to conversion of dynamic nonlinear estimation problem to dynamic linear estimation followed by static nonlinear fusion. In this work, we derive the discrete temporal evolution equation of the pseudo state vector, defined by the converted Doppler (the productive of target true range and range rate) and its first derivative, for the constant turn (CT) motion. The resulted linear state equation allows using of linear Kalman filter to extract information from the pseudo state of a target moves with constant speed and constant turn rate. The method is referred to as FKF. This proposed CT model is demonstrated by assessing the performance of the CDMKF and FKF. Comparative results show the superior performance of the proposed method especially in challenging scenario with large position measurement errors.

I. INTRODUCTION

The radar gives us the polar values like range, range rate, and angles of a particular target. These parameters are taken during its motion. Estimating with these measurements is somewhat complicated. So these parameters are converted as polar to Cartesian. During this conversion, we get some errors. Then the Cartesian components errors in the converted measurements are correlated with each other are explored in [4, 5, 9, 11, 13, and 14]. In this approach we have to consider the measurements of the target state estimation in a nonlinear fashion, which results the mixed coordinate filter [7], [8].

These measured terms results are considered to compare with the first two moment approximations which are presented here. The new converted measurement Kalman filter (CMKF) [14], is having estimation errors, which are compatible with the calculated covariance of the measured terms. The EKF is different from this method, because it is consistent only for small errors. So that the CMKF is having the correct covariance, it processes all the target measurements with a gain, which is nearly optimal and gives smaller errors compared with the EKF [3]. In the moderately accurate sensors, the EKF performs very poorly in tracking the target at long range for RMS azimuth error of 1.5 degrees or more [10]. But the CMKF [12] is consistent for 10° RMS azimuth error also.

In this paper to rectify these shortcomings, a new method is proposed. In the proposed method, the use of the nonlinear recursive filtering methods is avoided during the processing of Doppler measurements [6]. In the first one, a pseudo state vector is considered, which is the existing converted Doppler measurements of the target are linear functions and then they are constructed. These pseudo state vectors consist of the converted Doppler measurements and its derivatives [7], [8]. The pseudo state equations are derived from the measurements and proven to be linear in two commonly used target motion models. The other one is done by using extended Kalman filter (EKF) presented in [3, 6, 8, 10, 12, and 13]. One model is the constant velocity (CV) and the other one is constant acceleration (CA) models. Now adding the constant turn rate (CT) method to these converted Doppler measurement Kalman filter (CDMKF), is proposed to estimate the pseudo states [7]. This is also used for filter the noise in the converted Doppler measurements kalman filter. Finally, the CDMKF is combined with that constant rate method [14, 13] to construct a new filter which gives a new state estimator called as Fused Kalman Filter (FKF).

II. PROBLEM DESCRIPTION

In Cartesian coordinates target’s parameters are considered by depending on the conversion measurements of the target from polar coordinates to Cartesian. It is modeled as:
Where $X(k)$ is the state vector consisting of target's position components and corresponding target's velocity components along $x$ and $y$ directions, respectively, at every time step $k$. If a moving target is considered, the state vector can be taken by other components such as acceleration. Here, $\Phi(k) \in \mathbb{R}^{n \times n}$ is the target's state transition matrix, $v(k)$ is zero-mean Gaussian random process noise with covariance $Q(k)$, and $\Gamma(k)$ is noise gain matrix [2].

If we considered a 2D Doppler radar, which is assumed to report measurements of moving targets in polar coordinates, including range, range rate (Doppler) and angle presented in [3, 14, 13].

The measurement equation can be expressed as

$$z(k) = [r_m(k), \theta_m(k), \hat{r}(k)]^T$$

$$= h[x(k)] + w(k) = [r(k, \theta(k), \hat{r}(k))]^T + w(k)$$

$$= h[x(k)] + w(k) = [r(k, \theta(k), \hat{r}(k))]^T + w(k)$$

(2)

Where $r(k) = \sqrt{x^2(k) + y^2(k)}$

$$\theta(k) = \tan^{-1}[y(k)/x(k)]$$

(4)

$$\hat{r}(k) = \frac{x(k)x(k) + y(k)y(k)}{\sqrt{x^2(k) + y^2(k)}}$$

(5)

$$w(k) = [\hat{r}(k), \hat{\theta}(k), \hat{\theta}](k)$$

(6)

Normally the measured range and bearing of the target [14] are considered by taking the true range $r$ and bearing $\theta$ as

$$r_m = r + \hat{r} \quad \theta_m = \theta + \hat{\theta}$$

(7)

III. MEASUREMENT CONVERSION

The errors like range $\hat{r}$ and bearing $\hat{\theta}$ are taken to get independent with zero mean and standard deviations presented in [1]. These polar measurements are converted in to Cartesian coordinate measurements by using the following conversion techniques [14]

$$x_m = r_m \cos \theta_m; \quad y_m = r_m \sin \theta_m$$

(8)

The errors can be found by expanding these terms

$$x_m = x + \hat{x} = (r + \hat{r}) \cos(\theta + \hat{\theta});$$

$$y_m = y + \hat{y} = (r + \hat{r})\sin(\theta + \hat{\theta})$$

(9)

A conversion of the Doppler measurements is also made in this paper to yield the converted Doppler measurements as [7]:

$$\eta_c(k) = \eta_m(k) + \hat{\eta}(k)$$

(10)

Where $\eta(k)$ is the converted Doppler (i.e., the product of range and range rate), given by

$$\eta(k) = x(k)x(k) + y(k)y(k)$$

(11)

and $\hat{\eta}(k)$ is the error in the converted Doppler Measurement $\eta_c(k)$, [10].

$$z^c = \begin{bmatrix} x_m^c \\ y_m^c \end{bmatrix} = \begin{bmatrix} r_m \cos \theta_m \\ r_m \sin \theta_m \end{bmatrix} - \mu_c$$

Where the elements of $\mu_c$ are taken from (14) and the average covariance of the converted measurements is $R_a$ with elements (15).

Similarly, one can get the bias and variance of the converted Doppler measurements as [8]

$$\mu_n(k) = \rho \sigma_r \sigma_r$$

(12)

$$R_{\eta\eta}(k) = \eta_m^2(k) + \sigma_r^2 \eta_m^2(k) + 3(1 + \rho^2) \sigma_r^2 \sigma_r^2 +$$

$$2 \eta_m(k) \hat{\eta}_m(k) \rho \sigma_r \sigma_r$$

(13)

The debiased converted position measurements are given as

$$z^c_\hat{\eta}(k) = \eta_c(k) - \mu_\eta(k)$$

(14)

The covariance between the converted position measurements and the converted Doppler measurements can be given as [8]

$$R_{\eta\eta}(k) = \begin{bmatrix} r_m \cos \theta_m \\ r_m \sin \theta_m \end{bmatrix} - \mu_c$$

IV. CONVERTED DOPPLER AND POSITION MEASUREMENT KALMAN FILTER

A. Derivation of the CDMKF

Considering the above measurements the measurement equation can be taken as

$$z^c_\eta(k) = H_\eta(k)\eta(k) + w_\eta(k)$$

(16)

Where $z^c_\eta(k)$ the debiased converted Doppler measurement. The measurement matrix is
Where $\eta$ is the dimension. $w_{h}(k)$ is zero mean Gaussian noise. By considering these measurements the predicted state is

$$\hat{\eta}(k + 1, k) = \Phi_{\eta}\hat{\eta}(k, k) + G_{h}(k)$$  \hspace{1cm} (18)

Subtracting the measurements above equation with the term in the state equation it results

$$= \Phi_{\eta}\hat{\eta}(k) + \Gamma_{r}\nu_{r}(k) + \Gamma_{v}\nu_{v}(k)$$  \hspace{1cm} (19)

The predicted measurement can be easily obtained by the expected value at time step k as

$$\hat{z}_{e}^{T}(k + 1) = \Phi_{\eta}\hat{\eta}(k + 1, k)$$  \hspace{1cm} (20)

From these the measurement prediction covariance can be considered as

$$S_{\eta}(k + 1) = E[\hat{z}_{e}^{T}(k + 1, k)\hat{z}_{e}^{T}(k + 1, k)^{T}]$$

$$= \Phi_{\eta}P_{\eta}(k + 1, k)H_{\eta}^{T} + R_{\eta}^{M}(k)$$  \hspace{1cm} (21)

Taking the covariance between the pseudostate and measurement equations we get

$$P_{nz}(k + 1, k) = E[\hat{\eta}(k + 1, k)\hat{z}_{e}^{T}(k + 1, k)^{T}]$$

$$= \Phi_{\eta}P_{\eta}(k + 1, k)H_{\eta}^{T}$$  \hspace{1cm} (22)

The filter gain of the system is

$$K_{\eta}(k + 1) = P_{nz}(k + 1, k)S_{\eta}(k + 1)^{-1}$$

$$= P_{\eta}(k + 1, k)H_{\eta}^{T}S_{\eta}(k + 1)^{-1}$$  \hspace{1cm} (23)

Now considering the updated pseudostate estimate from the measurements is given as

$$\hat{\eta}(k + 1, k + 1) = \hat{\eta}(k + 1, k) + K_{\eta}(k + 1) \times [z_{e}^{T}(k + 1) - \hat{z}_{e}^{T}(k + 1, k)]$$  \hspace{1cm} (24)

And from all these calculations the updated covariance of the pseudostate is considered at time step k+1 is

$$P_{\eta}(k + 1, k + 1) = P_{\eta}(k + 1, k) - P_{nz}(k + 1, k)$$

$$\times S_{\eta}(k + 1)^{-1}P_{nz}(k + 1, k)^{T}$$  \hspace{1cm} (25)

V. FUSED KALMAN FILTER

The CDMKF provides a new method to exploit Doppler measurements. But the resulting pseudo states from the CDMKF are quadratic [7], [8], not linear, in Cartesian states. Additional processing is needed to extract the final target states from the pseudo states. The Cartesian states can be provided by the CPMKF, which is used along with the CDMKF, leading to a new tracking filtering approach, the FKF [13].

Fig. 1 illustrates the structure of the SF-CMKF. The original sensor measurements (i.e., range, Doppler, and angle) are divided into two parts to be processed separately by two linear filters first.
The prior mean of the state to be estimated is
\[ \bar{x}(k+1) = E[x(k+1)/x_p(k+1, k+1)] \] (33)
Debiased converted measurement is
\[ z(k+1) = \eta(k+1) - \hat{\eta}(k+1, k+1) \] (34)
The covariance between the states to be estimated and the measurement is
\[ P_{xz} \triangleq E[(x-\bar{x})(z-\bar{z})^T] \] (35)
The covariance of the measurement is
\[ P_{zz} \triangleq E[(z-\bar{z})(z-\bar{z})^T] \] (36)
The static nonlinear estimation equation is obtained as
\[ \hat{X} = \hat{x}_p + P_{xz}(P_{zz})^{-1}(\eta - \bar{z}) \] (37)

VI. CONSTANT TURN (CT) METHOD

All these methods were designed for linear motion of the target i.e. target moving with constant velocity on a straight line, in which case the position estimation error is small. In this work, a new method is aimed for, which shows superior performance when the position estimation error is more. In order to attain this, the target is considered to move with constant turn rate. This method may be referred to as parallel structure converted measurement kalman filter (PS-CMKF). In this work, the constant turn (CT), another commonly used maneuvering motion model is explored. This method presents the definition of the pseudo-state vector and the detailed derivation of the pseudo state equation for the CT motion as well as the parallel filtering procedure. The effectiveness of the constant turn (CT) model is demonstrated by evaluating the performance of the CDMKF. The superiority of the parallel structure converted measurement kalman filter (PS-CMKF) over other commonly used techniques is presented in the comparative simulations. In Cartesian coordinates, the motion with constant speed and constant turn rate can be expressed by a continuous time equation as
\[ \begin{bmatrix} \dot{x}(k) \\ \dot{y}(k) \end{bmatrix} = \begin{bmatrix} -w\dot{y}(k) \\ w\dot{x}(k) \end{bmatrix} \] (38)
Where \( w= \) Known turn rate, \( \dot{x}(k) \) and \( \dot{y}(k) \) are velocities in x and y coordinates, \( \ddot{x}(k) \) and \( \ddot{y}(k) \) are accelerations in x and y coordinates. The continuous-time form of the converted Doppler is expressed as
\[ \eta(k) = x(k)\ddot{x}(k) + y(k)\ddot{y}(k) \] (39)
For constant turn (CT) motion, the derivatives of Doppler measurement up to the second-order can be obtained as
\[ \hat{\eta}(k) = \dot{x}(k) + \dot{y}(k) + w[y(k)\ddot{x}(k) - x(k)\ddot{y}(k)] \]
\[ \ddot{\eta}(k) = -w^2[x(k)\dddot{x}(k) + y(k)\dddot{y}(k)] = -w^2\eta(k) \] (40)
Where \( \hat{\eta}(k) \) and \( \ddot{\eta}(k) \) are first and second order derivatives of \( \eta(k) \).

The state vector is defined as
\[ \begin{bmatrix} x^d(k) \end{bmatrix} = [\eta(k), \dot{\eta}(k)] \] (41)
The continuous-time pseudo-state equation is
\[ \dot{x}^d(k) = Ax^d(k) + D\ddot{\eta}^d(k) \] (42)
Where
The discrete time equation with sampling interval \( T \) is
\[ A = \begin{bmatrix} 0 & 1 \\ -w^2 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \] (43)
Where the discrete-time state is
\[ x_k^d = \begin{bmatrix} \eta_k \\ \dot{\eta}_k \end{bmatrix} = d(x_k) = \begin{bmatrix} x_k\dot{x}_k + y_k\dot{y}_k \\ \dot{x}_k^2 + \dot{y}_k^2 + w(y_k\ddot{x}_k + x_k\ddot{y}_k) \end{bmatrix} \]
\[ F^d = \begin{bmatrix} \cos wT & \sin wT / w \\ -w\sin wT & \cos wT \end{bmatrix} \]
\[ v_k^d = \int_0^T e^{A(T-\tau)} D\ddot{\eta}^d(kT + \tau)d\tau \]

Filtering Structure
The standard Kalman filtering procedure of CDMKF is expressed by pseudo equation is
\[ \begin{bmatrix} \hat{x}_{p,k}^d, P_{x,k}^d, k_{x,k}^d \end{bmatrix} = KF_d(z_{x,k}, \hat{x}_{p,k-1}^d, P_{x,k-1}^d, F^d, Q^d, H^d, R^d) \] (44)
The standard Kalman filtering procedure of CPMKF is expressed as
\[
\hat{x}_{k|k}, P_{k|k}, K_{k|k} = KF_p (z_{k|k}^c, \hat{x}_{k|k-1}, P_{k|k-1}, F_p, Q_p, H_p, R_p)
\]  
(45)

The cross covariance between the estimation errors of the CPMKF and CDMKF can be recursively updated by
\[
P_{k|k}^{pd} = [I - K_{k}^{p} H_{k}^{p}] (z_{k|k}^{c-p}, \hat{x}_{k|k-1}, P_{k|k-1}^{p}, F_{k}, Q_{k}, H_{k}, R_{k}^{p})
\]  
(46)

So by using the above considerations the updated state estimate of the system is
\[
[\hat{x}_k, P_k] = ([\hat{x}_{k|k}^{d}, P_{k|k}^{d}], \hat{x}_{k|k}, P_{k|k}^{p})
\]  
(47)

VII. SIMULATION RESULTS

Considering the target starting and moving with two trajectories which gives the effectiveness of the CDMKF and CPMKF methods in the forms nearly constant velocity trajectory and also nearly constant acceleration trajectory which are starting at (10km, 10km) and the target moves with a speed of 10m/s heading to 60 degrees. In the second scenario the acceleration is of 0.2 \text{m/s}^2. The process noise of the target is assumed to be zero-mean white Gaussian noise with standard deviation 0.001m/s. The sensor is located at origin (0km, 0km) and the sampling interval is \(T=1\)s. The standard deviations of target’s range, azimuth, and Doppler measurements are taken as, \(\sigma_r=50\text{m}, \sigma_{\theta}=2.5\text{deg}, \sigma_v =0.1\text{m/s}\). The correlation coefficient of the target between range and bearing is taken as \(\rho=0\). Simulations are performed here, over 200 time steps with the 50 Monte Carlo experiments. The motion of the targets and the root mean squared (RMS) errors of the following methods are considered and are shown in fig 2, fig 3, fig 4, and fig 5. The results are shown with Root Mean Square (RMS) error for the NCV and NCA trajectories, respectively. The effectiveness of the PS-CMKF as tracking filter is illustrated by comparing the performance of this method with that of the sequential non linear filtering method based on the sequential extended kalman filter (SEKF) and the sequential filtering approach with the sequential unscented kalman filter (UKF). The three tracking filters are having approximately the same RMSEs.

VIII. CONCLUSION

In this paper, the use of nonlinear recursive filtering approaches is avoided while processing the Doppler measurements. A linear filter, the converted Doppler measurement kalman filter (CDMKF), is proposed to estimate the pseudo states and filter the noise in the converted Doppler measurements. CDMKF can be used to operate along with the CPMKF to construct a new state estimator, Fused Kalman Filter (FKF). Cartesian state and pseudo state estimates are produced by CPMKF and CDMKF, respectively, and are then combined by a static estimator to obtain final state estimates.
The non-linearity of the pseudo states is quadratic and is handled by expanding the pseudo states up to the second term around the estimated states of the CPMKF. This investigated the constant turn (CT) model for the converted Doppler measurement Kalman filter. The pseudo state vector was defined by converted Doppler and its first derivative according to the CT model in Cartesian coordinates. The discrete temporal evolution equation of the pseudo state and the filtering procedure were derived. Simulations were performed to demonstrate the validity of the presented CT model by assessing the performance of the CDMKF and FKF. Comparative results showed the robustness to position measurement errors of the FKF.

REFERENCES