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Butterfly Effect and Lorenz Equations

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***Abstract*— In this paper, "Butterfly effect and Lorenz equations", we proceed to obtain Lorenz's attractor using the parametric values taken by Lorenz which are σ = 10, b = 8/3 while Reduced Rayleigh number to be r = 28. The following plots are the outcome of 10,000 iterations performed taking the dimensionless time increment h = 0.01. The plots are trajectories in phase space of X – Y, Y – Z and Z – X. These plots result in butterfly attractor which is a classical example of 'order emerging from chaos'. We solve those equations numerically applying Heun's method and plot the phase trajectories for different values of the parameters and obtain the outputs using MATLAB.**

***Keywords*— Butterfly effect, Heun's method, butterfly attractor, Lorenz's attractor, chaos, Lorenz equations, thermal diffusivities**

I. INTRODUCTION

In his classical paper entitled "Deterministic Non-periodic Flow", Edward N. Lorenz introduced a set of three ordinary differential equations. As he was interested in weather forecasting and while doing the experiment found that certain hydro-dynamical systems exhibit irregular behavior and do not tend to repeat their previous patterns.

After the suitable *de-dimensionalization* of the variables, following ODE's were proposed:

$$\dot{X} = -\sigma X + \sigma Y,$$
$$\dot{Y} = -XZ + rX - Y,$$
$$\dot{Z} = XY - bZ \qquad \text{-------- Set (1)}$$

Here the dot denotes the derivative with respect to the dimensionless time. The Prandtl number which is the ratio of the viscous to thermal diffusivities and depends on the nature of the fluid under consideration. The parameter *b* is a function of the aspect ratio and *r* is so called Reduced Rayleigh number. In fact, if the system is broken into small ideal cells called 'Rayleigh – Bénard' cells, *r* is the measure of temperature difference across such a cell in scaled units.

Now, in these equations, *X* is proportional to the intensity of the convection motion and *Y* is proportional to the temperature difference between the upward and downward currents. The part of fluid at higher temperature will rise and which is at lower temperature descends. The variable *Z* gives the departure of the vertical temperature profile with linearity. The last equation of the set (1) also suggests that the strongest gradients occur near boundaries.

Lorenz numerically solved (1) for specific values of the parameters and found the chaotic attractor known as The Butterfly attractor. This is a classical example of 'order emerging from chaos'. We again solve those equations numerically applying Heun's method and plot the phase trajectories for different values of the parameters. We proceed to obtain Lorenz's attractor using the parametric values taken by Lorenz which are σ = 10, b = 8/3 while *Reduced* Rayleigh number to be *r* = 28. The following plots are the trajectories in phase space of *X* – *Y*, *Y* – *Z* and *Z* – *X*. These plots are outcome of 10,000 iterations performed taking the dimensionless time increment *h = 0.01* .

II. GRAPHS GENERATED

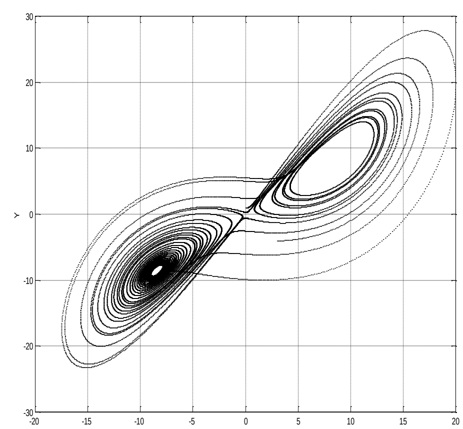

Figure 1: Projections on X – Y plane (for 30,000 iterations)

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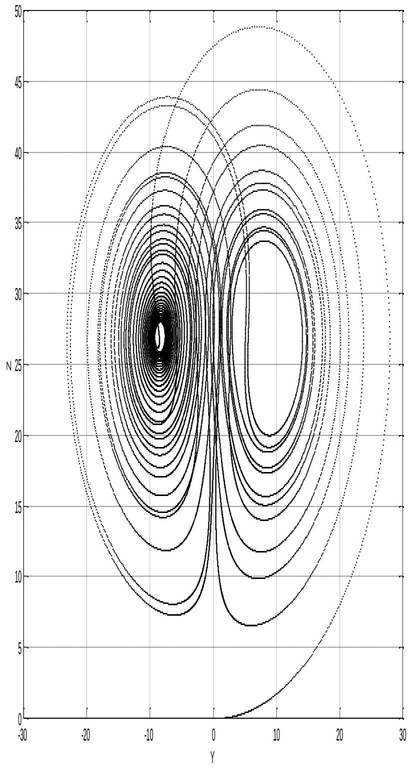

Figure 2 : Projections on the Y-Z plane (for 30,000 iterations)

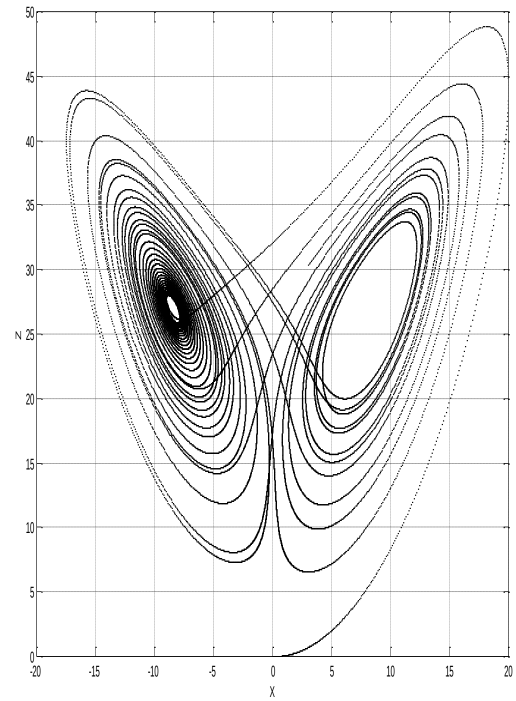

Figure 3: Projections on X – Z plane (for 30,000 iterations)

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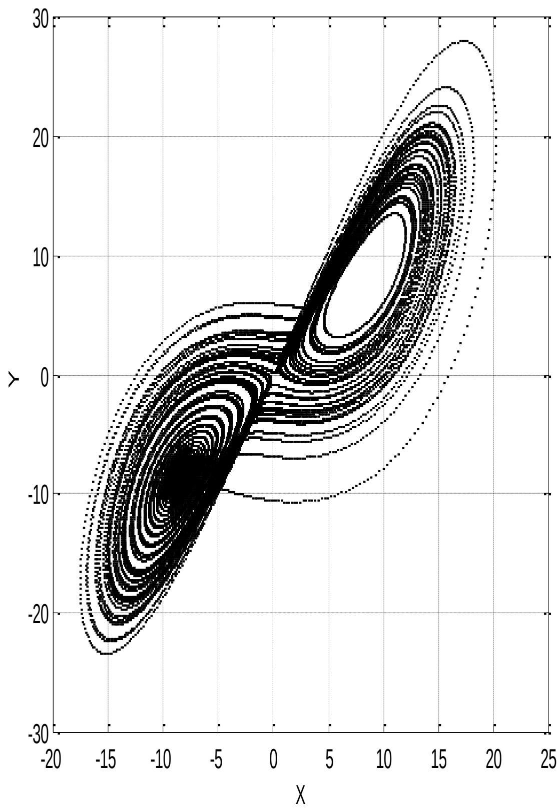

Figure 4: Projection in the X – Y plane taking h = 0.002 (for 30,000 iterations)

III. CONCLUSION

This butterfly attractor is a classical example of 'order emerging from chaos'. We have solved those equations numerically applying Heun's method and plotted the phase trajectories for different values of the parameters. We proceeded to obtain Lorenz's attractor using the parametric values taken by Lorenz which are σ = 10, b = 8/3 while *Reduced* Rayleigh number is taken to be r = 28. The following plots are the trajectories in phase space of $X - Y$, $Y - Z$ and $Z - X$. These plots are outcome of 10,000 iterations or more performed taking the dimensionless time increment $h = 0.01$ resulting in the butterfly attractor.

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