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Stability of the Libration Points of A Rotating Photo-Gravitatinal Triaxial Ellipsoid

P. K. Choudhary

Department of Mathematics, Dr. J. Mishra College, Muzaffarpur – 842001, B. R. A. Bihar University, Muzaffarpur, Bihar, India

Abstract**-- Stability of the libration points (the equilibrium points) of a rotating photogravitational triaxial ellipsoid has been studied in the strict sense. In the plane of parameters, the region of stability and instability of the libration points of rotating photogravitational triaxial ellipsoid are obtained.**

It is observed that the necessary condition of stability are satisfied according to the equations of the 1st approximation, the equilibrium points will be stable in the strict sense for majority values of the parameters.

Keywords**-- Stability, Rotating photogravitational triaxial ellipsoid.**

I. Introduction

Peter Musen (1960) made an analytic study of the influence of the solar radiation pressure on the motion of an artificial satellite. Musen, Bryant and Bailie (1960) could conclude that the effect in the ordinary case was so small that the two years of tracking observation of Vanguard-1 were required to reveal its pressure and at the end of that time the magnitude of the pressure could still be determined with a precision of only 30 percent. Later on, Musen under the resonance conditions showed that substantial orbit perturbations may be noted over intervals of several months. The perigee distance seen to decrease at the rate of 1 to 2 kilometer per day, so that the life time of satellite become considerably shorter than it would without this effect. The long periodic terms make the principal contribution to the orbit perturbations. Realizing the importance of many authors like Radzieveskii (1950), Kunitsyn and Perezhogin (1978), Schuerman (1970), Chernikov (1970), Simmons et. al. (1985), Kumar V and Choudhary, R.K. (1986, 1987, 1988, 1990), Sharma R.K. (1987) studied the restricted three body problem assuming the effect of radiation pressure to be significant.

The practical utility of the problem has attracted the attentions of many of 20th century space-scientists. A great deal of research works have been dedicated to the study of motion and stability of the libration points of rotating triaxial ellipsoid. Among them the work of Batrokov (1957) has its own importance.

He studied that in a co-ordinate system referred to gravitational triaxial ellipsoid rotating at a constant angular velocity, the differential equation of a mass point is the neibourhood of the ellipsoid admit of particulars solutions corresponding to the four libration points located along extensions of the major and minor axes of the equations equatorial plane of the ellipsoid. Abalakin V.K. (1957) studied the stability of the libration points on the extended minor axes are, to first approximation, stable position of relative equilibrium in the sense of Lyapunov (the necessary condition of stability are satisfied). Zhuravlev S.G. (1972) investigated the stability of the equilibrium points (the libration points) in the problem of mass point in the neighbourhood of a rotating triaxial ellipsoid in the strict sense. He observed that the libration points of ellipsoid, the form and dynamical characteristics of which are close of the planet of the solar system, are stable. He also ((1973) studied the same problem in a degenerate case and followed that equilibrium points in the degenerate case are stable in the strict sense. He (1975) studied the same problem in the three dimensional case and observed that the libration points on the extended minor axis of the equatorial plane of the ellipsoid, are stable for most (in the sense of Lebesgue) initial conditions, excluding two resonance cases previsously shown to be unstable in the plane case. Our purpose in this paper is investigating stability of the equilibrium points of a rotating photogravitational triaxial ellipsoid.

II. Equations Of Perturbed Motion

Let us consider a mass point is moving in the gravitational field of a rotating (with a constant angular velocity ω) the photogravitational triaxial ellipsoid of a mass M. We choose a rectangular co-ordinate system Oxyz with its origin at the centre of mass of the photogravitational ellipsoid, such that its axes coincide with the ellipsoid axes of inertia, with OZ axis corresponding to the rotational axis.

The differential equations of motion for a mass point in a co-ordinate system rotating at the constant angular velocity ω have the form

$$\ddot{x} - 2\omega\dot{y} - \omega^2 x = \frac{\partial u}{\partial x}$$

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$$\ddot{y} - 2\omega\dot{x} - \omega^2 y = \frac{\partial u}{\partial y}$$

$$\ddot{z} = \frac{\partial u}{\partial z} \qquad \text{.................. (1)}$$

where

$$U = f(1-q)\left\{\frac{M}{r} + \sum_{k=2}^{\infty}\frac{Q_k(x,y,z)}{r^{2k+1}}\right\} \qquad \text{.................. (2)}$$

In which f denotes the gravitational constant, q is the mass reduction factor, the force of radiation is given by

$$F = F_g - F_p = F_g\left(1 - \frac{F_p}{F_g}\right) = (1-q)F_g$$

Where

F_g is the gravitational attraction force,

F_p is the radiation pressure force,

q is the mass reduction factor.

Here it is assumed that gravitational prevails i.e. (1-q)>0 and 0<q<<1. Poynting-Roberston drag effect is ignrored, M is the mass of the ellipsoid, $r = (x^2 + y^2 + z^2)^{1/2}$, and Q_k are homogeneous polynomials of degree k with respect to x, y, z.

For example

$$Q_2 = \frac{1}{2}\left[(B+C-2A)x^2 + (A+C-2B)y^2 + (B+A-2C)z^2\right] \qquad \text{.................. (3)}$$

Where A, B and C are moments of inertial of the ellipsoid of second degree. For the triaxial ellipsoid which is close to a sphere the polynomial Q_2 can be written in the form -

$$Q_2 = M\left(\gamma x^2 + \mu y^2 + \nu z^2\right)$$

Where γ, μ and ν are small quantities. Where γ, μ, ν describes the deviation of the triaxial ellisoid from a sphere of equal volume; they satisfy the condition $\gamma + \mu + \nu = 0$.

The differential equations (1) with the force function (2) have particular solution corresponding to the libration points with co-ordinates

$$\left.\begin{aligned} L_1(L_3)&: x_0 = \pm a_0\left(1 + \frac{\gamma}{a_0^2} + ...\right), \quad y_0 = 0, z_0 = 0 \\ L_2(L_4)&: y_0 = \pm a_0\left(1 + \frac{\mu}{a_0^2} + ...\right), \quad x_0 = 0, z_0 = 0 \end{aligned}\right\} \qquad \text{.................. (4)}$$

Where

$$a_0 = \left\{fM(1-q)\omega^{-2}\right\}^{1/3} \qquad \text{.................. (5)}$$

Let us examine the stability of these equilibrium points and at the same time examine a plane case of motion.

Putting

$$\xi = x - x_0$$

$$\eta = y - y_0 \qquad \text{.................. (6)}$$

In equation (1), equations of perturbed motion of a mass point in a neighbourhood of the equilibrium point L_1, with take the following form

$$\ddot{\xi}-2\omega\dot{\eta}-\omega^2\xi=\frac{\partial u}{\partial \xi} \qquad \ddot{\eta}-2\omega\dot{\xi}-\omega^2\eta=\frac{\partial u}{\partial \eta}$$

Where

$$U=\left[\frac{1}{2}\left(a_1\xi^2+b_1\eta^2\right)+\frac{1}{3}a_2\xi^3+a_3\xi\eta^2+\frac{1}{4}\left(a_4\xi^4+b_4\eta^4\right)+\frac{1}{2}a_5\xi^2\eta^2\right] \quad \text{.................. (7)}$$

and co-efficients *a* and *b* are given below

$$a_1=2(A_1+\gamma B_1)$$

$$b_1=-(A_1-2\mu B_1)$$

$$a_2=-3(A_2+\gamma B_2)$$

$$a_3=\frac{1}{2}[3A_2+2B_2(\gamma-\mu)]$$

$$a_4=4[A_3+2\gamma B_3]$$

$$b_4=\frac{1}{2}[3A_3+4B_3(\gamma-\mu)]$$

$$a_5=-3[2A_3+B_3(3\gamma-2\mu)]$$

$$A_1=fM(1-q)\left(\frac{1}{x_0^3}+\frac{5\gamma}{x_0^5}+........\right)$$

$$B_1=fM(1-q)\left(\frac{1}{x_0^5}+........\right)$$

$$A_2=fM(1-q)\left(\frac{1}{x_0^4}+\frac{5\gamma}{x_0^6}........\right)$$

$$B_2=fM(1-q)\left(\frac{5}{x_0^6}+..........\right)$$

$$A_3=fM(1-q)\left(\frac{1}{x_0^5}+\frac{5\gamma}{x_0^7}........\right)$$

$$B_3=fM(1-q)\left(\frac{5}{x_0^7}+..........\right)$$

The characteristic equation of the system (7) can be written as

$$\lambda^4+\left(1-\frac{4\gamma+2\mu}{a_0^2}\right)\lambda^2+\frac{6(\mu-\gamma)}{a_0^2}=0 \quad \text{.................. (8)}$$

Equations (8) has two pair of roots.

$$\lambda_{1,2}=\pm i\omega$$

$$\lambda_{3,4} = \pm i\omega\sqrt{\frac{6(\mu-\gamma)}{a_0^2}}$$

It is observed that whole findings the roots, we considered only members of the 1st order in relation to quantities.

$$\sqrt{\frac{|\gamma|}{a_0^2}}, \quad \sqrt{\frac{|\mu|}{a_0^2}}$$

Consequently, in a case when $\mu > \gamma$ the characteristic equation (8) has two pairs of purely imaginary roots and therefore the necessary conditions of stability are satisfied.

When $\mu < \gamma$, the characteristic equation has two purely imaginary roots and two real roots. One of which will be positive and therefore sufficient conditions of unstability are satisfied.

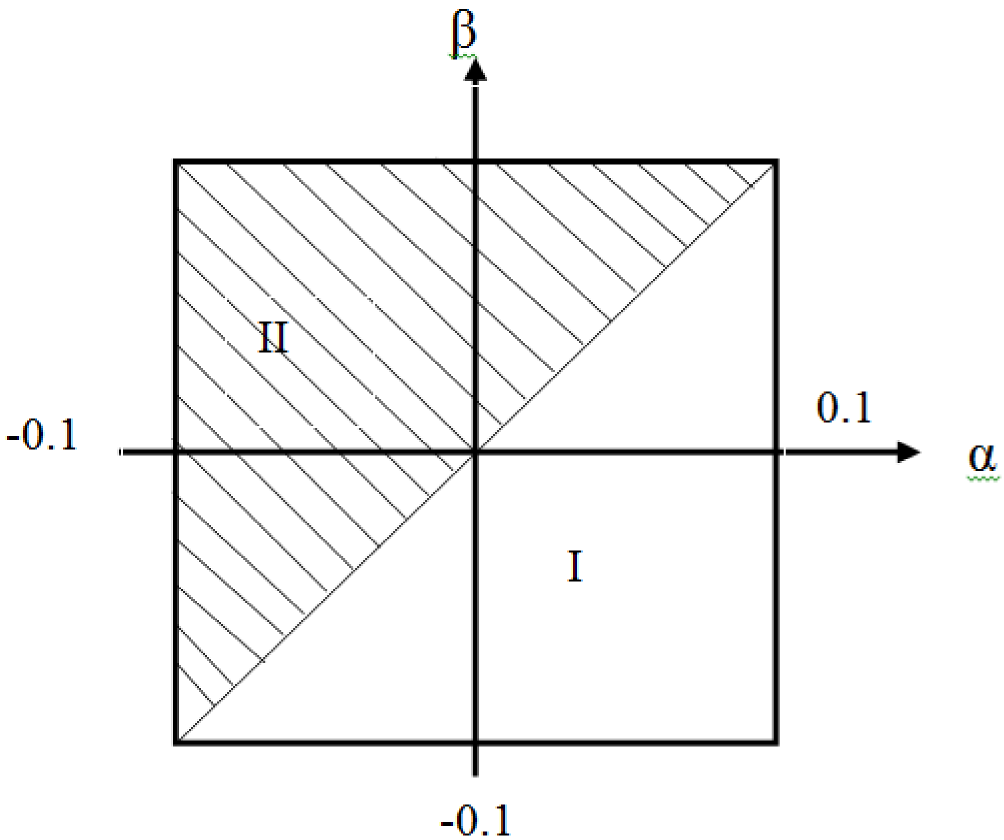

Fig. 1 : The regions of stability (i) and unstability (ii) of the solutions $L_1(L_3)$.

Considering in figure-1 on the plane of parameters $\alpha = \frac{\mu}{a_0^2}$ and $\beta = \frac{\gamma}{a_0^2}$ are shown (region(1)) where the necessary conditions of stability of the equilibrium points are satisfied, and (region-II), where the sufficient conditions of unstability are satisfied.

Showing that in region-I, according to the equations of the first approximation the necessary conditions of stability are satisfied, the equilibrium points will be stable in the strict sense for majority values of parameters α and β.

The above results satisfy the Zhuravlev (1972) results when we put q = 0.

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