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Equations of Translational-Rotational Motion of A Spheroidal Satellite in A Photogravitational Restricted Three Body Problem

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***Abstract*-- Equations of translational-rotational motion of a spheroidal satellite in a photogravitational restricted three body problem have been derived. It is observed that the analysis of Translational-Rotational motion on a spheroidal satellite in their gravitational field assuming that the sun is being perturbed by Photogravitational effect is bound to present realistic picture of the motion.**

***Keywords*-- Translational-Rotational, Photogravitational, Restricted three body.**

I. INTRODUCTION

Equations of Translational-Rotational motion of a spheroidal satellite in the gravitational field of two massing bodies having spherical symmetry were derived by Kondurar and Shinkarik (1972) in the case of circular restricted three body problem. The libration points in the generalised restricted three body problem have been investigated by choudhary R.K. (1977). However, they were restricted to the consideration of gravitational effects of the two massing bodies on the spheroidal satellite having infinitesimal mass. Presently most of the satellites are being launched in our solar system and hence the analysis of its motion in sun-planet system is most realistic. The sun being a radiating body, it is worth considering that the more massive primary sun is the source of radiation. Radzievskii (1950,1953) investigated the restricted three body problem taking into account the light pressure and analysed the location of equilibrium points for the translation motions of a point mass.

Chernikov (1970) investigated the linear stability of plannar equilibrium points of restricted three body taking into account photogravitational effect of the sun. Kunitsyn, A.L. Perizhogin, A.A. (1978) studied the linear stability of the triangular equilibrium points of the photogravitational circular restricted three body problem. Simmon's et. al. (1985) analysed the problem of existence of equilibrium points and their stability accounting for the radiation pressure of both the primaries for the translatory motions. Sharma, R.K. (1987) investigated the linear stability of libration points of the photogravitational restricted three body problem when the smaller primary is an oblate spheroid.

The sun which is the source of radiation is almost spherical and the planets may be considered to be approximately having spherical structure. The solar radiation pressure changes with the distance by the same law as the gravitational attraction force and acts opposite to it. Hence, it is relevant to consider that the result of action of solar radiation pressure force will lead to reducing the effect mass of the sun. Let F_g be the gravitational force of the sun and F_p be the solar radiation pressure force. The sun's resultant force acting on any particle is given by

$$F = F_g - F_p = F_g\left(1 - \frac{F_p}{F_g}\right) = qF_g \quad \text{where } q = 1 - \frac{F_p}{F_g}$$

is the mass reduction factor, constant for given particle. Obviously, the effect of the solar radiation pressure is to bring in a perturbation in gravitational attraction force by the constrant factor q. This constant factor can be expressed in forms of particle radius a, density δ and solar radiation pressure efficiency factor k (in C.G.S. unit) as :

$$q = 1 - \frac{5.6\times 10^{-5}}{a\delta}k$$

Assumption that q is constant amounts to neglecting the solar radiation flood fluctuations and shadow effect of the planet. The constraint q is positive when the gravitation prevails and negative if radiation pressure prevails. It is further assumed that the effect of radiation is to modify the magnitude of the inverse square law of force of the finite luminous mass acting on the infinitesimal mass, whereas the orbit of the finite masses is unaffected by the radiation pressure.

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II. Equations Of Motion

Let three bodies M_1, M_2 and *M* of mass m_1, m_2 and *m* respectively. Let us assume that *m* is so small in comparison of m_1 and m_2 that it has no effect on the motions of M_1 and M_2. Further more, assume that m_1, m_2 have a uniform spherical mass distribution so that they behave as point masses with regard to their gravitational effects on other bodies and m_1 the heaviest of the two is the source of radiation.

Finally, let assume that M_1 and M_2 are describing circular orbit about their mass centre in Newtonian reference frame with constant angular velocity n^2 where

$$n^2 = \frac{k^2(m_1+m_2)}{R^3} \qquad \text{.............. (1)}$$

Let us consider right handed rotating barrycentre co-ordinate system OXYZ with origin O at the centre of mass O of the finite bodies M_1 and M_2. X' directed along the line joining the masses, Y' axis along the transversal in the rotational plane and rotating with angular velocity *n* and Z' axis being orthogonal to the orbital plane forming a right handed system.

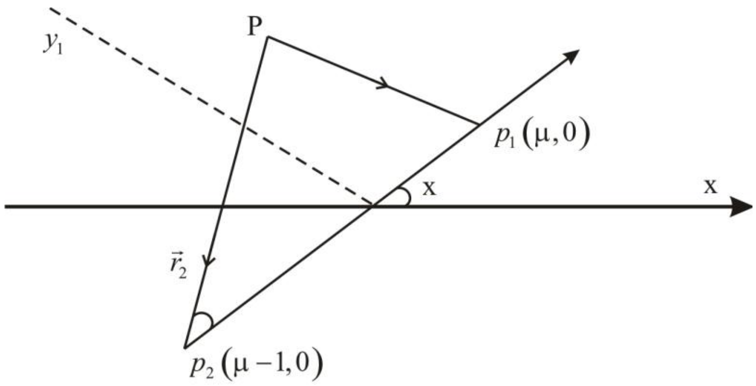

Fig. (1)

The position of the centrre of mass of the infenitesimal mass *m* is given by co-ordinate *XYZ* and radius vectors from m_i to *m* denoted by (*i* =1, 2) respectively. With the view to determine orientation of spheroidal satellite having infinetesimal mass, let us introduce orbital frame of reference $\xi_i\,(i=1,2,3)$ with origin G at the centre of mass infenitesimal mass *m* axis ξ, in radial direction joining O and G, and ξ_2 tangential to the orbit G in its direction of motion and ξ_3 normal to the orbital plane. Body axes $X_i\,(i=1,2,3)$ are obtained from axis $\xi_i\,(i=1,2,3)$ by means of three independent rotations in the following two ways.

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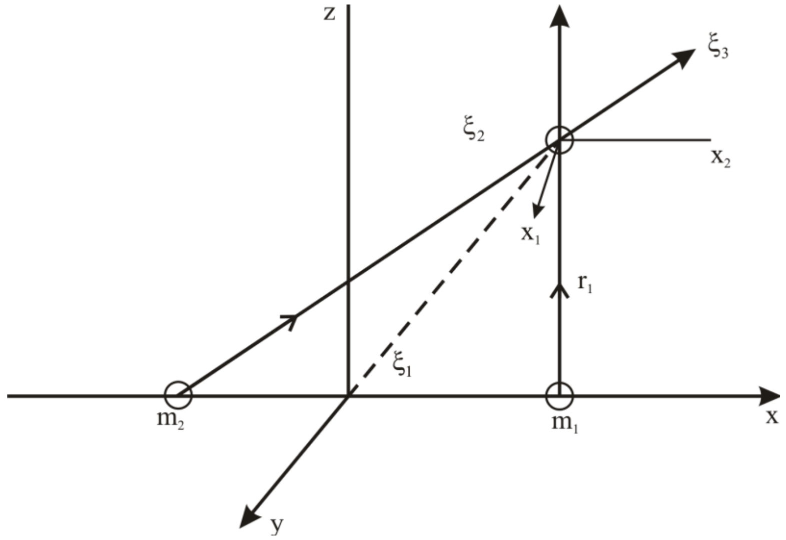

Fig. - 2

Firstly, the rotations in fig. (2) are θ_2 about ξ_2, θ_1 about η_1 and ϕ about ξ_3.

In the second case fig. (3) the rotations are given by familiar Eulerian angles ψ about ξ_3, θ about η_1, and ϕ about ξ_3. The relations between the two axis x_i and ξ_i can be given in terms of direction cosines l_{ij} by the matrix equation.

$$\{x\} = [l]\,\{a\} \qquad ....(2)$$

Thus the direction cosines between axis x_i and ξ_i have the expressions.

$$l_{11} = C\theta_2 C\phi + S\theta_2 S\phi_1, S\phi = C\psi C\phi - S\psi C\theta S\phi$$

$$l_{12} = C\theta_1 S\phi \qquad = S\psi C\phi + C\psi C\theta S\phi$$

$$l_{13} = S\theta_2 S\phi + C\theta_2 S\theta_1 S\phi \;= S\theta S\phi$$

$$l_{21} = -C\theta_2 S\phi + S\theta_2 S\phi_1 C\phi = -C\psi S\phi - S\psi C\theta C\phi$$

$$l_{22} = C\theta_1 C\phi \qquad = -S\psi S\phi + C\psi C\theta C\phi$$

$$l_{23} = S\theta_2 S\phi + S\phi + C\theta_2 S\theta_1 C\phi \;= S\theta C\phi$$

$$l_{31} = S\theta_2 C\theta_1 \qquad = S\phi S\theta$$

$$l_{32} = -S\theta_1 \qquad = -C\psi S\phi$$

$$l_{33} = C\theta_2 C\theta_1 \qquad = C\theta \qquad ....(3)$$

Where $S\theta$ and $C\theta$ denote $Sin\theta$ and $Cos\theta$ respectively.

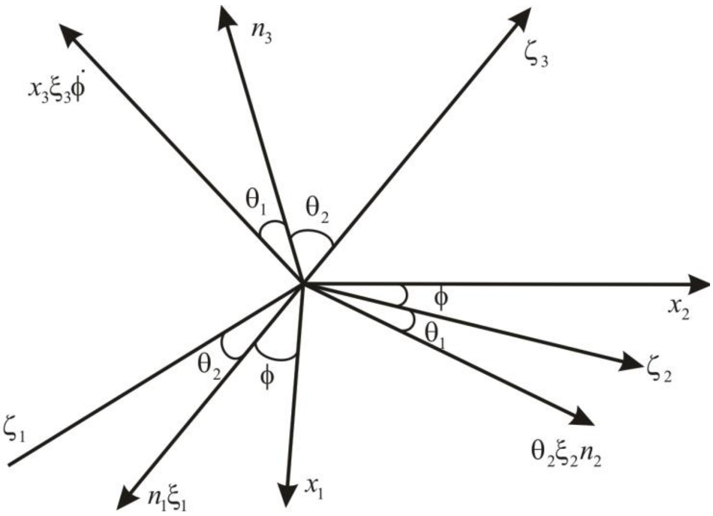

Fig.(3)

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The satellite being axially symmetric body it is advantageous to work with the axis ζ_1, ζ_2, ζ_3. The direction cosines between ζ_i and $\xi_i\,(i=1,2,3)$ are obtained by putting $\phi=0$ in eqns. (3).

The angular velocity components Ω_i of the body M about these axis are:

$$\Omega_1=\omega_1;\quad \Omega_2=\omega_2;\quad \Omega_3=\omega_3+\dot{\phi} \qquad \text{.................. (4)}$$

where ω_1, ω_2, ω_3 are angular velocities of the system $\zeta_i\,(i=1,2,3)$ and are easily obtained as

$$\left.\begin{aligned}\omega_1&=-nS\theta_2+\dot{\theta}_1 &&= \dot{\theta}\\ \omega_2&=-nC\theta_2S\theta_1+\dot{\theta}_2C\theta_1 &&= (n+\dot{\psi})S\theta\\ \omega_3&=-nC\theta_2C\theta_1-\dot{\theta}_2S\theta_1 &&= (n+\dot{\psi})C\theta\end{aligned}\right\} \qquad \text{.................. (5)}$$

In the rotating barrycentre co-ordinate system with generalised co-ordinates XYZ, θ, ϕ, ψ or XYZ, θ_1, θ_2, θ_3 the Lagragian of the problem can be easily written as

$$\bar{L}=T+\bar{U}$$

Where T is the kinetic energy and $\bar{U}$ is the potential energy.

The kinetic energy T is given by

$$T=\frac{1}{2}m\left[\left(\dot{X}-ny\right)^2+\left(\dot{Y}+nx\right)^2+\dot{Z}^2\right]+\frac{1}{2}\left(\bar{A}\omega_1^2+\bar{B}\omega_2^2+\bar{C}\omega_3^2\right) \text{................. (6)}$$

The force function $\bar{U}$ is given by

$$\bar{U}=\bar{U}_1+\bar{U}_2 \qquad \text{.................. (7)}$$

where

$$\bar{U}_1=k^2mm_1q\left[\frac{1}{R_1}+\frac{\bar{A}\bar{\sigma}}{2R_1^3}\left(1-3\gamma_1^2\right)\right] \qquad \text{................. (8)}$$

$$\bar{U}_2=k^2mm_2\left[\frac{1}{R_2}+\frac{\bar{A}\bar{\sigma}}{2R_2^3}\left(1-3\gamma_2^2\right)\right] \qquad \text{.................. (9)}$$

$$\bar{\sigma}=\frac{\bar{C}-\bar{A}}{\bar{A}}\left(-1\le 0\le 1\right) \qquad \text{................. (10)}$$

Let us use the non-dimensional variables

$$\frac{m_2}{M}=\mu,\ \frac{m_1}{M}=1-\mu,\quad M=m_1+m_2$$

$$R_1=Lr_1,\qquad R_2=Lr_2 \qquad \text{................ (11)}$$

$$x=\frac{X}{L_0},\ y=\frac{Y}{L_0},\ \tau=\frac{Z}{L_0}$$

$$r_1=\frac{R_1}{L_0},\ r_2=\frac{R_2}{L_0},\ \tau=nt$$

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$$x_1 = \frac{X_1}{L_0},\ y_1 = \frac{Y_1}{L_0},\ n^2 = \frac{k^2(m_1+m_2)}{L_0^3}$$

$$\bar{A} = mL_0^2 A,\ \bar{C} = mL_0^2 C$$

L_0 being the distance between the primaries and A, C are non-dimensional forms of moment of inertia of the oblate spheroid, the mass ratio of the small primary to the total mass.

The Lagrangian $\bar{L}$ is reduced to

$$\bar{L} = \frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\left(\dot{y}x-y\dot{x}\right)+\frac{1}{2}\left(x^2+y^2\right)+$$

$$+\frac{1}{2}\left[A\left\{\left(S\theta_2-\dot{\theta}_1\right)^2+\left(C\theta_2 S\theta_1+\dot{\theta}_2 C\theta_1\right)^2\right\}+C\left(C\theta_2 C\theta_1-\dot{\theta}_2 S\theta_1+\dot{\phi}^2\right)\right]+\left[U_1+U_2\right] \qquad \text{..................(12)}$$

where $L = \dfrac{\bar{L}}{mn^2L_0^2}$, here (.) denotes the differentiation with respect to non-dimensional time τ .

$$L = \frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\left(x\dot{y}-y\dot{x}\right)+\frac{1}{2}\left(x^2+y^2\right)+$$

$$\frac{1}{2}\left[A\left\{\theta^2+\left(1+\psi\right)^2 S\theta_2\right\}+C\left\{\left(1+\dot{\psi}^2\right)\left(\theta+\dot{\phi}\right)^2\right\}\right]+\left[U_1+U_2\right] \qquad \text{................ (12')}$$

where

$$U_1 = (1-\mu)q\left[\frac{1}{r_1}+\frac{A\sigma}{2r_1^3}\left(1-3\gamma_1^2\right)\right] \qquad \text{................. (13)}$$

$$U_2 = \mu\left[\frac{1}{r_2}+\frac{A\sigma}{2r_2^3}\left(1-3\gamma_2^2\right)\right] \qquad \text{................. (14)}$$

$$\sigma = \frac{C-A}{A}\left(-1\le 6\le 1\right) \qquad \text{................. (15)}$$

$$\gamma_i = \frac{1}{r_i}\left[\left(x-x_i\right)l_{31}+yl_{32}+zl_{33}\right] \qquad \text{................ (16)}$$

$$l_{31} = S\theta_2 C\theta_1 = S\psi S\theta$$

$$l_{32} = -S\theta_1 - = -C\psi S\theta \qquad \text{................ (17)}$$

$$l_{33} = C\theta_2 C\theta_1 = C\theta$$

$$\mu_1 = 1-\mu,\ \ \mu_2=\mu,\ \ x_1=\mu,\ \ x_2=\mu-1,\ r_1^2=\left(x-\mu\right)^2+y^2+z^2$$

$$r_2^2 = \left(x-\mu+1\right)^2+y^2+z^2 \qquad \text{................ (18)}$$

γ_i being cosine of the angle between the radius vector r_i and the axis of symmetry of rotation of the satellite.

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III. Routhian Function For The Problem

On, inspection of the Lagrangian we observed that ϕ is not presents in the Lagrangian and hence it is ignorable co-ordinate.

Hence, the corresponding conjugate momenta $\frac{\partial L}{\partial \dot{\phi}}$ is a constant.

Thus,

$$\frac{\partial L}{\partial \dot{\phi}} = C\left[(1+\dot{\psi})C\theta + \dot{\phi}\right] = Cr_2$$

$$= C\left(C\theta_2 C\theta_1 - \dot{\theta}_2 S\theta_1 + \dot{\phi}\right) = Cr_0 \qquad \text{................. (19)}$$

r_0 being constant.

Hence, we obtain

$$C\theta_2 C\theta_1 - \dot{\theta}_2 S\theta_1 + \dot{\phi} = (1+\dot{\psi})C\theta + \dot{\phi} = r_0\,(cons.) \qquad \text{................. (20)}$$

Eliminating ignorable co-ordinate Routhian function R can be easily obtained as

$$R = \left[\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\frac{1}{2}\left(\dot{\theta}_1^2+\dot{\theta}_2^2 C\theta_1^2\right)\right]+\left[(x\dot{y}-y\dot{x})+\dot{\theta}_2\left(AC\theta_1 C\theta_2 S\theta_1 - Cr_0 S\theta_1\right)-A\dot{\theta}_1 S\theta_2\right]$$

$$+\left[\frac{1}{2}\left(x^2+y^2\right)+\frac{1}{2}A\left(S\theta_2^2+S\theta_1^2 C\theta_2^2\right)+Cr_0 C\theta_1 C\theta_2-\frac{1}{2}\left(r_0^2+U\right)\right] \qquad \text{................. (21)}$$

$$R = \left[\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\frac{1}{2}\dot{\theta}^2+\frac{1}{2}S\theta^2\dot{\psi}^2\right]+\left[x\dot{y}-y\dot{x}+A\dot{\psi}S\dot{\theta}^2+Cr_0\dot{\psi}C\theta\right]$$

$$+\left[\frac{1}{2}\left(x^2+y^2\right)+\frac{1}{2}AS\theta^2+Cr_0 C\theta-\frac{1}{2}Cr_0^2+U\right] \qquad \text{................ (22)}$$

where

$$U = U_1 + U_2$$

Obviously, the original system having six degrees of freedoms have been reduced to a problem with five degrees of freedom. The sixth co-ordinate and its derivative is obtained by quadrature with the help of the equation. (19).

Hence, an attempt will be made to obtain the equations of motion with the help of the Routhian functions obtained.

The Hamiltonian for the reduced system is given by

$$H = -R + \sum \frac{\partial R}{\partial v_i}\dot{q}_i = T_2 - T_0 - U \qquad \text{................. (23)}$$

As R does not contain time τ , the Jocobi's integral

$$H = T_2 - T_0 - U = h \qquad \text{................ (24)}$$

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Exists for the problem. Hence Jacobi's integral is given by

$$\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\frac{1}{2}\left(\dot{\theta}_1^2+\dot{\theta}_2^2 C\theta_1^2\right)-\frac{1}{2}A\left(S\theta_2^2+S\theta_1^2 C\theta_2^2\right)-Cr_0 C\theta_1 C\theta_2- \quad -$$

$$\frac{1}{2}\left(x^2+y^2\right)-U+\frac{1}{2}Cr_0^2=h \qquad \text{................ (25)}$$

$$\Rightarrow \frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\frac{1}{2}A\left(\dot{\theta}^2+\psi S\theta^2\right)-\frac{1}{2}\left(x^2+y^2\right)-\frac{1}{2}AS\theta^2-Cr_0 C\theta-U+\frac{1}{2}Cr_0^2=h \qquad \text{............. (26)}$$

This can be put in the form

$$T_2+\nu=h \qquad \text{................ (27)}$$

where ν is the reduced potential energy given by

$$\nu=-\frac{1}{2}\left(x^2+y^2\right)-\frac{1}{2}A\left(S\theta_2^2+S\theta_1^2 C\theta_2^2\right)-Cr_0 C\theta_1 C\theta_2-U+\frac{1}{2}Cr_0^2 \qquad \text{................ (28)}$$

$$\nu=-\frac{1}{2}\left(x^2+y^2\right)-\frac{1}{2}AS\theta_2^2-Cr_0 C\theta-U+\frac{1}{2}Cr_0^2 \qquad \text{................ (29)}$$

It is not difficult to obtain the equations of motion of the system in the following form :

$$\ddot{x}-2\dot{y}=-\frac{\partial \nu}{\partial x}$$

$$\ddot{y}-2\dot{x}=-\frac{\partial \nu}{\partial y}$$

$$\ddot{z}=-\frac{\partial \nu}{\partial z} \qquad \text{................ (30)}$$

$$A\ddot{\theta}-A\left(\dot{\psi}^2+2\dot{\psi}\right)S\theta C\theta+\dot{\psi}Cr_0 S\theta=-\frac{\partial \nu}{\partial \theta}$$

$$AS\theta^2\ddot{\psi}+2\left(\dot{\psi}+1\right)\dot{\theta}AS\theta C\theta-Cr_0\dot{\theta}S\theta=-\frac{\partial \nu}{\partial \psi}$$

or $$A\ddot{\theta}_1+A\dot{\theta}_2^2 S\theta_1 C\theta_1-2A\dot{\theta}_2 C\theta_2 C\theta_1^2+Cr_0\dot{\theta}_2 C\theta_1=-\frac{\partial \nu}{\partial \theta_1}$$

$$AC\theta_1^2\ddot{\theta}_2-2A\dot{\theta}_1\dot{\theta}_2 S\theta_1 C\theta_1+2A\dot{\theta}_1 C\theta_1^2 C\theta_2-Cr_0\dot{\theta}_1 C\theta_1=-\frac{\partial \nu}{\partial \theta_2}$$

where

$$\nu=-\frac{1}{2}\left(x^2+y^2\right)-\frac{1}{2}\left(S\theta_2^2+S\theta_1^2 C\theta_2^2\right)-Cr_0 C\theta_1 C\theta_2-U+\frac{1}{2}Cr_0^2$$

$$=-\frac{1}{2}\left(x^2+y^2\right)-\frac{1}{2}AS\theta^2-Cr_0 C\theta-U+\frac{1}{2}Cr_0^2$$

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