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Certain Investigation On The Higher Dimensional LRS Bianchi Type-I Cosmological Model Universe Interacting With Perfect Fluid In Lyra Manifold

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Abstract - **In this paper, we have considered a locally rotationally symmetric (LRS) Bianchi type - I cosmological model universe interacting with perfect fluid in the context of Lyra geometry. Exact solution of field equations are obtained with constant deceleration parameter models of the universe and certain physical assumptions in different cases. We have discussed different distribution like empty, dust or coherent matter, stiff fluid, Radiation universe and Matter distribution in inter-nebular space. The Lyra's geometry, which is a generalization of Weyl's geometry (removing the defect of non-integrability of length transfer), gravitational theory and the cosmology based on this geometry have been reviewed. Here, we have presented a higher dimensional (N-dimensional) plane symmetric metric in Lyra's geometry with constant deceleration parameter. Further, cosmological models with constant deceleration parameter models have been discussed. We discuss a model universe with different situations, by solving the modified Einstein field equation within the framework of Lyra geometry. This model is suitable for an early stage of the universe, that is, before the universe underwent the compactification transition. It is observed that our model universe is always an isotropic one. Many previously known solutions are contained here in as a particular case. Some physical and kinematical features of the models are also discussed.**

Keywords - **Higher dimensional LRS Bianchi type cosmological model, Weyl's geometry, Lyra's geometry, Deceleration parameter, coherent matter, stiff matter.**

I. INTRODUCTION

Various researchers have focused their mind to study of higher dimensional cosmological models within the framework of Lyra's geometry and other theories of relativity. Kaluza and Klein [9] tried to unify gravity with electromagnetic interaction by introducing an extra dimension. Their theory is an extension of Einstein's general relativity to five dimensions. According to Chodos and Detweiler [10], the present four-dimensional stage of the universe could have been preceded by a higher-dimensional stage, which at a later period becomes four-dimensional in the sense that the extra dimensions contract to unobserved planckian length scale due to dynamical contraction.

In view of the emergence of superstring theory as the most promising theory developed so far, having the potential to lead us to a step closer towards unification of four forces, studies in higher-dimensional cosmology have obtained renewed importance inspiring a host of workers to enter into this field of study. Already a number of important solutions of Einstein's equations in higher dimensions have been obtained by many authors [3-8] constructed higher dimensional cosmological models in general theory of relativity. Subsequently Sen [47], Sen and Dunn [48] suggested a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry.

Einstein developed his general theory of relativity, where gravitation is described in terms of geometry. Based on the cosmological principle, Einstein introduced the cosmological constant into his field equations in order to obtain a static model of the Universe, because without the cosmological term his field equations admit only non-static cosmological models for non-zero energy density. Later, Weyl [1] proposed a more general theory in which electromagnetism is also described geometrically. He showed how one can introduce a vector field in the Riemannian space-time with an intrinsic geometrical significance. But this theory was based on non-integrability of length transfer so that it had some unsatisfactory features, and hence this theory which is known as Weyl's geometry still today did not gain general acceptance. After having these concepts, Lyra [2] suggested a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry, by introducing a gauge function into the structure-less manifold, which removes the non-integrability condition of the length of a vector under parallel transport and a cosmological constant is naturally introduced from the geometry. Halford [3,4] pointed out that in the normal general relativistic treatment the constant displacement vector field φ_i in Lyra's geometry plays the role of cosmological constant and the scalar-tensor treatment based on Lyra's geometry predicts the same effect, within observational limits, as far as the classical solar system test are concerned (as in the Einstein's theory of relativity).

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In the past and recent years many prominent researchers like [5-27] have investigated and proposed different cosmological models and ideas of the Universe within the framework of Lyra's geometry and other theories of relativity in different context. But the main problem in Astrophysics is the discovery, about two decades ago, that our Universe expansion is accelerating, instead of slowing down as predicted by the Big Bang theory [28]. Observational evidence for accelerated expansion in the Universe has been growing during this period [29, 30, and 31]. Independent confirmation using observations of high red shift supernovae [32-37] along with observations of cosmic microwave background radiation (CMB) [38-40] and large scale structure [41] have made this result more acceptable to the community. In fact, the recent observations of Type SNeIa supernova, CMB anisotropies the large scale galaxies structures of universe and Sachs Wolf effects have led to the idea that our universe undergoes accelerated expansion at the present epoch tending to a de-Sitter space-time as predicted by inflation theory [42-46].Moreover, solutions of Einstein field equations in higher dimensional space times are believed to be of physical relevance possibly at extremely early times before the Universe underwent the compactification transitions. As a result, now the higher dimensional theory is receiving great attention in both cosmology and particle physics. Particle physicists and cosmologists predicted the existence of GUT (Grand Unified Theory). Using a suitable scalar field it was shown that the phase transitions on the early universe can give rise to such objects which are nothing but the topological knots in the vacuum expectation value of the scalar field and most of their energy is concentrated in a small region. As the necessity to study higher-dimensional space-time in this field aiming to unify gravity with other interactions the concept of extra dimension is relevant in cosmology [50-51]. In particular, for the early stage of the Universe and theoretically the present four dimensional stage of the Universe might have been preceded by a multi-dimensional stage.

So, in this paper we discuss the higher dimensional cosmological models in Lyra geometry by considering locally rotationally symmetric (LRS) Bianchi Type-I metric with the use of deceleration parameter and certain physical assumption to find out the solutions compatible with the observational facts. We discuss a model universe with different situations, by solving the modified Einstein field equation within the framework of Lyra geometry.

II. Field Equations And Their Solutions

Here we consider the five dimensional plane symmetric space-time in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2(dy^2 + dz^2) + C^2 d\zeta^2 \qquad (1)$$

With the convension $x^1 = x, x^2 = y, x^3 = z, x^4 = \zeta$, $x^5 = t$ where A, B and C are function of time ‘‘ t ’’ only. Here the extra coordinate is taken to be time-like.

Einstein field equations based on Lyra's geometry, which in normal gauge may be written as

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\varphi_i \varphi_j - \frac{3}{4} g_{ij}\varphi_k \varphi^k = -\chi T_{ij} \qquad (2)$$

where, φ_i is the displacement vector, c = 1, $8\pi G = \chi$ and other symbols have their usual meaning in the Riemannian geometry.

The N - dimensional time like displacement vector φ_i in (1) is defined as

$$\varphi_i = (0,0,0,0, \alpha(t),) \qquad (3)$$

The energy-momentum tensor is taken as

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \qquad (4)$$

Where p, ρ and u^i are pressure, energy density and five dimensional velocity vector of the fluid distribution, respectively. That is, u^i has a component

$$u^i = \left(\frac{1}{A}, 0,0,0,0\right) \qquad (5)$$

Also let, $x^i = \left(\frac{1}{A}, 0,0,0,0\right)$

Together with the coordinate satisfying

$$g_{ij} u^i u^j = -1 = -x^i x_i \text{ and } u^i x_i = 0 \qquad (6)$$

In co-moving coordinate system, we have from Eqn. (4)

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p \;;\quad T_5^5 = \rho$$

$$T_j^i = 0 \text{ for all } i \neq 0 \qquad (7)$$

Using Eqns. (3) - (7), the surviving field eqn. (2) for the metric in Eqn. (1) are obtained as

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{B}\dot{C}}{BC} - 2\frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} + \frac{3}{4}\alpha^2 = A^2 p \qquad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\alpha^2 = A^2 p \qquad (9)$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} - \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{3}{4}\alpha^2 = A^2 p \qquad (10)$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3}{4}\alpha^2 = -A^2 \rho \qquad (11)$$

Now from eqns. (9) and (10) we have

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{K}{B^2 C} \qquad (12)$$

where $K > 0$ is an integration constant.

Since the field equation (8)-(11) are highly non-linear, so in order to obtain the exact solution of Eqns. (8)-(11), we use the following scale transformations as used by [49].

$$A = e^a, B = e^b, C = e^c \ \& \ dt = AB^2CdT \tag{13}$$

Using transformations of Eqn. (13) in Eqns. (8) - (11) we have

$$2b'' + c'' - 4a'b' - 2b'c' - 2c'a' - b'^2 = pe^{2(2a+2b+c)} - \frac{3}{4}\alpha^2 e^{2(a+2b+c)} \tag{14}$$

$$a'' + b'' + c'' - 3a'b' - 2b'c' - 2c'a' - a'^2 - b'^2 = pe^{2(2a+2b+c)} - \frac{3}{4}\alpha^2 e^{2(a+2b+c)} \tag{15}$$

$$a'' + 2b'' - 4a'b' - 2b'c' - c'a' - a'^2 - b'^2 = pe^{2(2a+2b+c)} - \frac{3}{4}\alpha^2 e^{2(a+2b+c)} \tag{16}$$

$$2a'b' + 2b'c' + c'a' + b'^2 = -\rho e^{2(2a+2b+c)} + \frac{3}{4}\alpha^2 e^{2(a+2b+c)} \tag{17}$$

Where dashes denotes derivative with respect to time 'T'

Now solving Eqns. (14)-(16) we have

$$a = b = c \tag{18}$$

Therefore, from Eqn. (13) we have

$$A = B = C \tag{19}$$

By using Eqn. (19) in Eqn. (8) - (11) we have

$$3\frac{\ddot{A}}{A} + \frac{3}{4}\alpha^2 = A^2 p \tag{20}$$

$$6\frac{\dot{A}^2}{A^2} - \frac{3}{4}\alpha^2 = -A^2\rho \tag{21}$$

There are two independent equations involving four unknowns A, α, p and ρ. So, in order to get deterministic solutions of the above set of highly nonlinear Eqns. (20)-(21), we shall use the special law of variation of Hubble's parameter proposed by Bermann [13] that gives constant deceleration parameter as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{22}$$

Where q is a constant and

$$R = \left(\sqrt{A}B^3C^2\right)^{\frac{1}{4}} \tag{23}$$

is the overall scale factor.

Here the constant 'q' is taken as negative, so the model is an accelerating model of the universe.

Solving Eqn. (22) we have

$$R = (\beta t + \gamma)^{\frac{1}{q+1}} \tag{24}$$

Where, $\beta \neq 0, q + 1 \neq 0$ *and* γ are constant.

By using Eqn. (19) in Eqn. (24), we have

$$A = R^{8/11} \tag{25}$$

Therefore, from Eqns. (19), (24) and (25) we have

$$A = B = C = R^{8/11} = (\beta t + \gamma)^{\frac{8}{11(q+1)}} \tag{26}$$

Now we consider two cases describe below.

Case-I when α is a constant

From Eqns. (20) and (21) we have

$$p = \frac{3}{4}\frac{\alpha^2}{(\beta t+\gamma)^{\frac{16}{11(q+1)}}} - \frac{24}{121}\frac{\beta^2(11q+3)}{(q+1)^2(\beta t+\gamma)^{\frac{16}{11(q+1)}+2}} \tag{27}$$

$$\rho = \frac{3}{4}\frac{\alpha^2}{(\beta t+\gamma)^{\frac{16}{11(q+1)}}} - \frac{192}{121}\frac{\beta^2}{(q+1)^2(\beta t+\gamma)^{\frac{16}{11(q+1)}+2}} \tag{28}$$

Here, the integrating constant β and are to be chosen in such a way that ρ and p are non-negative.

Now, the metric in Eqn. (1) using Eqn. (28) can be written as

$$ds^2 = (\beta t+\gamma)^{\frac{16}{11(q+1)}}\{dx^2 + dy^2 + dz^2 + d\zeta^2 - dt^2\} \tag{29}$$

The above equation (29) together with Eqns. (27) and (28) will be the exact 5-D LRS Bianchi type-I perfect fluid cosmological model in Lyra Geometry when α is a constant.

Now, if we take $q = \alpha = 0$ then from Eqns. (27) and (28) we have

$$p = -\frac{24}{121}\frac{\beta^2}{(\beta t+\gamma)^{\frac{38}{11}}} \tag{30a}$$

$$\rho = -\frac{192}{121}\frac{\beta^2}{(\beta t+\gamma)^{\frac{38}{11}}} \tag{30b}$$

Since both ρ and the p are negative so from the above two equations we have

$$\rho = 8p$$

which satisfies the general equation of state:

$$p = \xi\rho$$

Case-II: When α is a function of t

There are two independent field equations, Eqns. (20) and (21), involving three unknowns ρ, p and α. So, in order to get deterministic solution we must have to assume a physical or mathematical condition amongst the unknowns. Here we consider the equation of state (i.e., physical condition) as

$$p = \xi\rho \tag{31}$$

Case II-(a): Empty universe [i.e., = ρ = 0]

Now we consider

$$p = \rho = 0$$

Putting these value in Eqn. (20 - 21), we have

$$\frac{3}{4}\alpha^2 = \frac{24}{121}\frac{\beta^2(11q+3)}{(q+1)^2(\beta t+\gamma)^2} \tag{32}$$

The above equation (29) together with equation (32) will constitute an exact 5-D LRS Bianchi type-I empty space model in Lyra geometry.

Case II-(b): Dust (or, in coherent matter) distribution [ξ = 0, i.e., p = 0 and $\rho \neq 0$]

When ξ = 0, then from Eqn. (31) we have

$$p = 0 \tag{33}$$

Putting $p = 0$ in Eqn. (20), we have

$$\frac{3}{4}\alpha^2 = \frac{24}{121}\frac{\beta^2(11q+3)}{(q+1)^2(\beta t+\gamma)^2} \tag{34}$$

Using Eqns. (34) in (21)

$$\rho = \frac{24}{121}\frac{\beta^2(11q-13)}{(q+1)^2(\beta t+\gamma)^{\frac{16}{11(q+1)}+2}} \tag{35}$$

The above equation (29) together with equations (33) - (35) will constitute an exact 5-D LRS Bianchi type-I coherent matter distribution model universe in Lyra geometry.

Case II-(c): Stiff (or, Zel'dovich) fluid distribution [i.e.; ξ = 1]

When ξ = 1 then from Eqn. (31) we have

$$p = \rho \tag{36}$$

when ξ =1 i.e., when p = ρ then we can see that it is not possible to find out a physically meaningful solution for the field equations.

Therefore, when α is a function of time t then Bianchi type-I cosmological stiff fluid universe does not exist in this theory.

Case II-(d): Disordered distribution of Radiation (or, Radiation Universe) [i.e.; $\xi = \frac{1}{3}$]

When $\xi = \frac{1}{3}$ then from Eqn. (31) we have

$$\rho = 3p \tag{37}$$

Using ρ = 3p in Eqns. (20) and (21) we have

$$p = \frac{12}{121}\frac{\beta^2(11q-13)}{(q+1)^2(\beta t+\gamma)^{\frac{16}{11(q+1)}+2}} \tag{38}$$

$$\rho = \frac{36}{121}\frac{\beta^2(11q-13)}{(q+1)^2(\beta t+\gamma)^{\frac{16}{11(q+1)}+2}} \tag{39}$$

Therefore, from Eqn. (20-21) we have

$$\frac{3}{4}\alpha^2 = \frac{12}{121}\frac{\beta^2(33q-7)}{(q+1)^2(\beta t+\gamma)^2} \tag{40}$$

Eqn. (29) together with Eqns. (37 - 40) will constitute an exact 5-D LRS Bianchi type-I radiating model universe in Lyra geometry.

Case II-(e): Matter distribution in inter-nebular space [i.e.; $\xi = \frac{2}{3}$]

When , $\xi = \frac{2}{3}$, then from Eqn. (31) we have

$$\rho = \frac{3}{2}p \tag{41}$$

Using $\rho = \frac{3}{2}p$, we have from Eqns. (20) and (21)

$$p = \frac{48}{121}\frac{\beta^2(11q-13)}{(q+1)^2(\beta t+\gamma)^{\frac{16}{11(q+1)}+2}} \tag{42}$$

$$\rho = \frac{72}{121}\frac{\beta^2(11q-13)}{(q+1)^2(\beta t+\gamma)^{\frac{16}{11(q+1)}+2}} \tag{43}$$

Therefore, from Eqn. (20-21)we have

$$\frac{3}{4}\alpha^2 = \frac{24}{121}\frac{\beta^2(33q-19)}{(q+1)^2(\beta t+\gamma)^2} \tag{44}$$

Eqn. (29) together with Eqns. (41- 44) will constitute an exact 5-D LRS Bianchi type-I cosmological model universe in the matter distribution in inter-nebular space in Lyra geometry.

In all the cases, II.(a)-(d) the reality condition $\rho > 0$ is obtained as

$$q > \frac{13}{11} \tag{45}$$

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Now in all the above four cases we see that the value of the deceleration parameter q > 0 i.e., our model is an accelerating one.

III. Physical And Geometrical Properties Of The Solutions

Here, the spatial volume V and the average scale factor *R(t)* for the Bianchi type-I plane symmetric metric (Eqn. (1)) defined by $V = R^4(t) = -g^{\frac{1}{2}} = \sqrt{A}B^3C^2 = A^{\frac{11}{2}}$ of the model are given by

$$V = (\beta t + \gamma)^{\frac{4}{q+1}} \tag{46}$$

$$R(t) = (\beta t + \gamma)^{\frac{1}{q+1}} \tag{47}$$

We observed that the volume V is increasing with the increase of time if q + 1 > 0 i.e., if q > -1 and the volume V is decreasing with the increase of time and tends to zero as t → ∞ if q + 1 < 0 i.e., if q < -1. Also the scale factor R is increasing with the increase of time if q+1 > 0 i.e., if q > -1 and the scale factor R is decreasing with the increase of time and tend to zero as t → ∞ if q + 1 < 0 i.e., if q < -1.

Also, the mean Hubble's parameter H is obtained as

$$H = \frac{\beta}{(q+1)(\beta t+\gamma)} \tag{48}$$

From the above Eqn. (48) it has been observed that in the initial stage, when

t = 0 we get

$$t = \frac{\beta}{(q+1)\,(\gamma)} \tag{49}$$

Again, the value of H decreases with the increase of time t and finally H becomes zero whenever t → ∞. Also the Hubble's parameter H becomes infinite whenever $q = -1 \ or \ t = -\frac{\gamma}{\beta}$.

The expansion factor θ calculated for the flow vector u^i is given by

$$\theta = u^i_{;i} = \frac{1}{A}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 4\frac{\dot{A}}{A^2}$$
$$= \frac{32\beta}{11(q+1)(\beta t+\gamma)^{\frac{8}{11(q+1)}+1}} \tag{50}$$

The model has a singularity at $t = -\frac{\gamma}{\beta}$ and the scalar expansion $\theta \to 0 \ as\ time\ t \to \infty \ if \ q > -\frac{1}{11}$.

The components of the shear scalar σ for the metric in Eqn. (1) are given by

$$\sigma_1^1 = \frac{1}{A}\left(\frac{\dot{A}}{A} - \frac{A\theta}{4}\right) \tag{51}$$

$$\sigma_2^2 = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{A\theta}{4}\right) \tag{52}$$

$$\sigma_3^3 = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{A\theta}{4}\right) \tag{53}$$

$$\sigma_4^4 = \frac{1}{A}\left(\frac{\dot{C}}{C} - \frac{A\theta}{4}\right) \tag{54}$$

$$\sigma_5^5 = 0 \tag{55}$$

Therefore, the shear scalar σ for the metric in Eqn. (1) is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2 + (\sigma_5^5)^2] = 0 \tag{56}$$

Since $\sigma^2 = 0$ so our model universe is shear free. Also since $\frac{\sigma}{\theta} = 0$ for all values of 't', so our model universe is always an isotropic one.

IV. Conclusion

In this paper, we have considered a LRS Bianchi type- I cosmological model universe interacting with perfect fluid in the context of Layra's geometry by using constant deceleration parameter. We have discussed different distributions like empty space, dust, stiff fluid, disordered distribution and Matter distribution in inter-nebular space and it is observed that our model universe is always an isotropic one.

- Our N-dimensional model is suitable for early stage of the universe and explains the different types of distributions of matter and for different type of symmetries of space time.

V. Future Prospects

The investigation on this topic can be further taken up in different directions:

- This topic has been a proliferation of works on higher dimensional cosmological model in Lyra geometry with time dependent displacement field.
- It is important in a natural way to make a search for exact solutions for constant deceleration parameter with different types of distributions of matter and for different type of symmetries of space time.
- This also helpful to provide the idea about study of physical situation at the early stages of the formation of the universe and isotropic behaviour.

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Nomenclature :

g_{ij} *fundamental tensor*
H *Hubble's parameter*
H_0 *constant of integration*
$\dot{H}$ *dot denotes differentiation with respect to time.*
p *pressure*
q *deceleration parameter*
K *constant of integration*
R_{ij} *Ricci tensor*
R(t) *length Scale*
T_{ij} *energy-momentum tensor*
u^i *N-dimensional velocity vector*

Greek symbols :

α *displacement field*
β *constant of integration*
$d\zeta^2$ *line element*
φ_i *displacement vector*
ρ *energy density*
ξ *arbitrary constant for equation of state.*