

# Dispersion Effects in Continuum Mechanics.

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**Abstract** - A more complete mathematical model of continuum mechanics briefly outlines. The influence of angular momentum of particles without structure arises at any gradients of physical parameters (density, speed, temperature). Angular momentum, delay and position of inertia center are investigated. Disturber of the ergotic law is discussed for the classic equations of continuous environment. The new method of calculation pressure and energy for multicomponent environment was suggested. Not symmetrical stress tensor is obtained as results of influence of angular momentum for continuous medium. The result of the impact of new effects may be the emergence of additional force, which is not taken into account in modern engineering calculations. The method for writing of interaction discretion and continuous mediums was suggested. Attention is paying on delay for processes discrete mediums. Analytical results are obtained for cases of large gradient. The nucleus of the Navier-Stokes equations is obtained. Equations by S.V. Vallander were obtained from the kinetic equation. The appearance of self-diffusion and thermal diffusion is substantiated.

**Keywords**-- Angular momentum, delay, conservation laws, non-symmetrical stress tensor, Boltzmann equations, Chapman-Enskog method, conjugate problem the Navie-Stokes, S.V. Vallander equations

## I. INTRODUCTION.

Briefly outlines a more complete mathematical model of continuum mechanics. The angular momentum of particles without structure arises at any gradients of physical parameters (density, speed, temperature). The position of axis of inertia in the elementary volume, the lag time and space are taking into account in this work. The latter is due to the definition of a derivative as the limit of the ratio of infinitely small values, although a medium is discrete and for rarefied gas time between collisions and the distance between the molecules are finite. Classical mechanics was formed under the assumptions that the elementary volume is closed and conditions is close to equilibrium; were considered the potential forces. Today the systems work, usually in the conditions of large gradients of physical parameters. Angular momentum gives emergence of additional forces that can play the role of small perturbations affecting the stability of the structure or destruction of the body. The resulting effects may affect in a critical and a near critical modes of aircraft, rockets, various devices, structures, as well as in some of the natural processes. The action of angular momentum, i.e. torque depends substantially on the axis of inertia (center of mass).

A center of inertia of volume element can important both its rotation about its center of inertia and about the involvement of formulation angular momentum. This has led to an incomplete accounting of the processes so this caused the appearance of this article. Now for consideration of the angular momentum the theory of brothers E., Cosserat, F. Cosserat and their modifications are used [1]. L. D. Landau, C de Groot, P. Mazur, I. Deyrmati, I. Prigogine, L. I. Sedov, A. A. Ishlinskii, D. D. Ivlev et al. point at the role of angular momentum. It is the most developed area in the momentum theory of elasticity (R. D. Mindlin, V. A. Palmov, A. G. Gorshkov, E. I. Starovoitov, A. V. Yarovoy, V. Levin, S. E. Kanaun, E. L. Aero, etc.). Of course, only some authors are mentioned here. Questions of delay in a rarefied gas on the basis of the kinetic equation were discussed in [2,3]; in a solid [4-6]. P. Ya. Polubarinova-Kochina, S.F. Averyanov, M.G. Andersen, T.P. Burt, J. Baer, L. Duckstein, D. Zaslavski, S. Irmei, R. J. Hanke and others in their studies drew attention to the importance of the problems of the theory of groundwater movement, the atmospheric physics boundary layer, heat and mass transfer in capillary-porous bodies. Any movement of an elementary volume of liquid at the moment can be considered as a result of the following motion: quasi solid motion that is translated with selecting pole, rotating motion around this pole and deformation motion. This theorem was proved by H. Gelmholtz. L. Prandtl formulated conception of hard plastic body as the theory of ideal plasticity. Usually, we do not take into account twist velocity. The angular momentum is responsible for the twist velocity. Thus, in the classical Newton mechanics we have four conservation laws: masses, liner momentum, energy and angular momentum. In continuous mechanics, we use only three first laws. However, the law of conservation of angular momentum requires a particular frame of reference and a particular radius vector from the origin to the elementary volume. In the classical approach, the law of conservation of angular momentum does not fit. For these formulations, actually, they postulated symmetry of the material frame of reference and, as a consequence, the symmetry of the stress tensor and the violation of the "continuity" of the environment. Mathematically, this fact follows from the choice of the conditions of equilibrium of forces as a condition of equilibrium. Selection of the conditions of equilibrium of momentum of forces leads to new formulations of equations. The kinetic theory does not save the situation.

The law of angular momentum is not implemented in the Boltzmann equation. Classical theory, the second viscosity, is predicted. But usually, we suppose that it is in need to take into consideration for molecules with inner of degree of freedom or for dense gas. Equations by S.V. Vallander were obtained us from the kinetic equation with inclusion of the self-diffusion and thermo-diffusion. These equations were earlier obtained by Vallander [7,8] from phenomenological considerations. The classical kinetic theory was used, described in the books [9-13]. A more detailed exposition of our suggested theory is contained in [14-17].

## II. THE GENERAL PART

The main function in mechanics is the Lagrangian recorded relative to the fixed pole. However, the center of inertia moves for many material systems. Consequently, the general form of the derivative of the Lagrange function for a moving system has the form

$$\frac{dL}{dt} = \sum_i \left[ \frac{\partial L}{\partial \dot{q}_i} \dot{q}_i + \frac{\partial L}{\partial \dot{a}} \dot{a} \right] + \sum_i \left[ \frac{\partial L}{\partial (q_i - a)} (\dot{q}_i - \dot{a}) + \frac{\partial L}{\partial (q_i - \dot{a})} (\ddot{q}_i - \ddot{a}) \right],$$

$$\mathbf{a} = \sum_i \frac{m_i \mathbf{r}_i}{m_i}, \text{ for electrical interaction} \quad \mathbf{a} = \sum_i \frac{e_i \mathbf{r}_i}{e_i}.$$

Derivative determines the force. In addition, uneven distribution of physical quantities gives rise to the additional force due to the appearance of the angular momentum. Result:  $\mathbf{F} = \mathbf{F}_0 + \frac{d\mathbf{M}}{dR_i}$ , where  $\mathbf{F}$  - force, acting on the particle,  $\mathbf{F}_0$  - the power without taking into account the angular momentum, the  $M$  - the moment of force acting on the particle,  $\mathbf{R}_i$ . Equations have a higher order than the classic. Consequently, additional boundary conditions should be set. For Boltzmann equation must set boundary condition for the flow. For the equations of a continuous medium, you can specify the components of the velocity at the outer edge of the boundary layer and the velocity and vorticity. The classical kinetic theory is presented in books. Based on the modification of the Lagrangian were derived modified Liouville equation and Boltzmann. It should be noted that the classical Boltzmann equation does not comply with the law of conservation of angular momentum. This is clearly seen if we multiply equation of speed on radius-vector of the particle to get momentum. Even with central interaction we get different values for the non-equilibrium conditions. From modified Boltzmann equation we obtain the modified equations for gas

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i} \left( x_i \frac{\partial \rho u_i}{\partial x_i} \right) = 0,$$

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho F_x + \frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{yx}}{\partial y} + \frac{\partial P_{zx}}{\partial z}$$

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \rho F_y + \frac{\partial P_{xy}}{\partial x} + \frac{\partial P_{yy}}{\partial y} + \frac{\partial P_{zy}}{\partial z}$$

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho F_z + \frac{\partial P_{xz}}{\partial x} + \frac{\partial P_{yz}}{\partial y} + \frac{\partial P_{zz}}{\partial z} \quad (1)$$

$$\frac{\partial}{\partial t} \rho \left( \frac{3}{2} RT + \frac{1}{2} u^2 \right) + \frac{\partial}{\partial x_j} \left[ \rho u_j \left( \frac{3}{2} RT + \frac{1}{2} u^2 \right) + u_k P_{kj} + q_j \right] +$$

$$+ \frac{\partial}{\partial x_i} x_i \frac{\partial}{\partial x_j} \left[ \rho u_j \left( \frac{3}{2} RT + \frac{1}{2} u^2 \right) + u_k P_{kj} + q_j \right] = 0$$

$$y \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) - z \left( \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} \right) + \sigma_{zy} - \sigma_{zy} = 0$$

$$x \left( \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} \right) - y \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} \right) + \sigma_{yx} - \sigma_{xy} = 0$$

$$x \left( \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) - z \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} \right) + \sigma_{zx} - \sigma_{xz} = 0$$

The last three equations make it possible to declare the symmetry of the stress tensor only when the equilibrium condition of forces is satisfied (the brackets vanish). These conditions, used in the classical theory, are a special case of fulfilling the more general equilibrium conditions presented in the set of equations. The last three equations serve to determine the degree of non symmetry of the stress tensor. The notation is standard:  $u, v, w$ -velocities in the Cartesian coordinates  $x, y, z$ ;  $F_x, F_y, F_z$  is the force;  $P$  with indices is the stress tensor,  $T$  is the temperature,  $R$  is the gas constant. The new equations can be obtained phenomenologically if one considers the possibility of rotating an elementary volume about the axis of inertia of an elementary volume or the rotation of the elementary volume itself. For the theory of elasticity, the equations are the same as classic theory, but their interpretation varies.

The latter follows from the fact that it is necessary to consider the general expansion of velocity around an axis passing through an arbitrary point as a representation of velocity at a point through the sum of translational and rotational velocities relative to the axis of inertia of the elementary volume. If we add our terms in the last three equations, taking into account the internal angular momentum, we obtain the law of conservation of the momentum for ferromagnets. The theories used on the basis the theory of Cosserat brothers [14] concern structural particles. However, the angular momentum leading to the appearance of an additional force does not contain new dimensions, which is manifested in the equations for particles without structure. Equations (1) are of higher order than classical ones. Consequently, additional boundary conditions should be set. For Boltzmann equation must set boundary condition for the flow. For the equations of a continuous medium, you can specify the components of the velocity at the outer edge of the boundary layer and the velocity and vorticity. We considered both options for the solution of the modified problem Falkner-Skan. The results led to conclusion about increasing the role of the vertical component of the velocity and the penetration of the vorticity inside the boundary layer. For the layer of the mixing with constant speed on the outer boundary, friction on the border of mixing increased, but the character of the flow remained the same. In the kinetic theory when considering the role of delay should deal with the question of what the experiment: measures the instantaneous values or averaged. If the experiment is dealing with averages, it is important to choose the time and the distance of averaging. If the delay time and relaxation are close, to take into account the delay is necessary. For example, when considering the organic molecules in lasers. In the rarefied gas unusual situation arises when for describing the derivatives we use the limit of the ratio increments of the function to the increment of the argument. It turns out that for recording the time derivative we have a finite mean free path and a finite time interval between collisions, consequently, we consider only high-speed components, as slow collisions have not time to occur. To clarify the situation, we use a simple account of the delay in the form of the second term of the Taylor series.

To clarify the situation, we can use a simple account of the delay as the second term of the Taylor series. It is desirable, of course, to use the theory of generalized functions. For extreme cases of maximum and minor differences the times of flight of the molecules and the departing molecules the core of the Navier-Stokes equation can be definite, through additional term (mean is given for additional term (mean free path of the molecules and the run up to and after the collision can be different ( $\tau, \tau', \lambda, \lambda'$ );  $n, n_1$  - appropriate density;  $f^0, f_1^0$  - distribution function):

$$\left(\tau - \frac{n}{n_1} \tau\right) \frac{\partial f^0}{\partial t} f_1^0 + \left(\frac{n}{n_1} \tau - \frac{n}{n_1} \tau\right) f^0 \frac{\partial f_1^0}{\partial t} + \left(\lambda - \frac{\xi'}{\xi} \lambda\right) \frac{\partial f^0}{\partial x} f_1^0 + \left(\frac{n_1}{n} \lambda - \frac{n}{n_1} \lambda\right) \frac{\partial f_1^0}{\partial x} f^0$$

The effect of the spatial inhomogeneity of molecular distributions is the effects predicted by S.V. Vallander, we obtained from the kinetic theory in [12]. Based on the received representations in solving the flow problem, it is proposed to abandon the solution of the Boltzmann equation in the entire region, confining itself to the region of several radii of interaction between gas molecules and a solid by the molecular dynamics method. In this region, the gas molecules do not collide. In the rest of the region, it is proposed to solve the Navier-Stokes equations. The obtained macro parameters are used to restore the Chapman-Enskog distribution function at the outer boundary of the layer. In this formulation, the problem is solved more simply and can be solved as self-adjoint. The hypothesis of the absence of collisions in the layer is verified and the influence of the distribution function profile on the process of interaction of gas atoms with a crystalline surface was determined. The initial formulation of the problem is related to modeling by the molecular dynamics method of the influence of the distribution function profile on the interaction process. The equilibrium distribution function is used with a directed gas velocity at the outer boundary of the layer. As examples of the influence of the moment for the theory of elasticity, the problems of Prandtl, Lamé, Kirsch, rods with various types of loading was chosen to illustrate the theory [16]. The lack of symmetry of the stress tensor and the shift of the inertia axis, which is not taken into account in modern theories, can make a significant contribution to the strength of engineering structures.

### III. S.V. VALLANDER EQUATIONS AND THE CHAPMAN -ENMSKOG METHOD FOR THE BOLTZMANN EQUATION.

A known solution of the Chapman-Enskog obtained using many approximations. On the other hand, in the classical laws of conservation, that we study in this part, the normal velocity component [18] is used as a main velocity.

$$\frac{\partial}{\partial t} \int_{\tau} \rho \delta \tau + \int_{\sigma} \rho \mathbf{V}_n \delta \sigma = \int_{\tau} \dot{M} \delta \tau. \text{ Consequently, } f(t + dt, x + (\xi \cdot \mathbf{n}) dt, \mathbf{x} + (\xi \cdot \boldsymbol{\tau}) dt), \xi_i + \frac{x_{0i}}{m} dt) dx d\xi = f(t, \mathbf{x}, \xi_i) dx d\xi + (\Delta^+ - \Delta^-) dx d\xi dt$$

As a result, we need to get the conservation law in the form of

$$\frac{\partial \rho}{\partial t} + \frac{\partial[(\rho \mathbf{u}) \cdot \mathbf{n} + (\rho \mathbf{u}) \cdot \boldsymbol{\tau}]}{\partial x_i} = 0.$$

Where  $\mathbf{n}$ ,  $\boldsymbol{\tau}$  - the unit vectors along the normal and tangential to the surface. If the Boltzmann equation is written out in the projections, more properly, in the arbitrary of the volume should be considered normal and tangential velocities. The velocity projections on the coordinate axes are used in the numerical analysis. Therefore, the error values are of the order of self-diffusion and thermal diffusion, which will be determined by the tangential components. Our target is to understand the process of self-diffusion and thermal-diffusion through the equilibrium distribution function and investigate the effect of small additions on the values of macroscopic parameters. The equilibrium function  $f_0 = n^0 \left(\frac{m}{2\pi kT^0}\right)^{3/2} e^{-\frac{m(\xi - \mathbf{u})^2}{2kT^0}}$ . Let  $\Delta$  is a small correction.

The behavior of the function we are interested in the effect of calculating the Hilbert hypothesis on macro parameters through the equilibrium distribution function.

For  $\boldsymbol{\rho} \cdot \mathbf{u}$

$$\frac{1}{(n + \Delta n)} \int (n + \Delta n + \dots) \left(\frac{m}{2(\pi k(T + \Delta T))}\right)^{3/2} e^{-\frac{m(\xi - \mathbf{u} - \Delta \mathbf{u})^2}{2k(T + \Delta T)}} d\xi \cdot \int (n + \Delta n + \dots) \xi \frac{m}{2\pi k(T + \Delta T)} e^{-\frac{m(\xi - \mathbf{u} - \Delta \mathbf{u})^2}{2k(T + \Delta T)}} d\xi.$$

We are using the formulas expansion in the series given that in both cases the first terms of the formulas for  $\boldsymbol{\rho} \cdot \mathbf{u}$  and  $(\rho u)$  match.

The difference between the approximations define with first-degree order and has a structure of the solution of the Chapman-Enskog.

$$\begin{aligned} & \frac{1}{n} \left(1 - \frac{\Delta n}{n} + \dots\right) \int \mathbf{n} \left(1 + \frac{\Delta n}{n} + \dots\right) \left(\frac{m}{2\pi kT}\right)^{3/2} \left(1 - \frac{3}{2} \frac{\Delta T}{T} \dots\right) e^{-\frac{m(\xi - \mathbf{u} - \Delta \mathbf{u})^2}{2k(T + \Delta T)}} d\xi = \\ & \frac{1}{n} \left(1 - \frac{\Delta n}{n} + \frac{\Delta n}{n} + \dots\right) \left(1 - \frac{3}{2} \frac{\Delta T}{T} \dots\right) \int \mathbf{n} \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m(\xi - \mathbf{u})^2}{2kT}} e^{-\frac{m(\xi - \mathbf{u})^2}{2kT} \left(-2(\xi - \mathbf{u})\Delta \mathbf{u} - \dots \frac{\Delta T}{T}\right)} d\xi = \\ & \frac{1}{n} \left(1 + \dots\right) \left(1 - \frac{3}{2} \frac{\Delta T}{T} \dots\right) \int \mathbf{n} \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m(\xi - \mathbf{u})^2}{2kT}} \left(1 - \frac{m(\xi - \mathbf{u})^2}{2kT^0} \left(-2(\xi - \mathbf{u})\Delta \mathbf{u} - \dots \frac{\Delta T}{T}\right)\right) d\xi \\ & \frac{m(\xi - \mathbf{u} - \Delta \mathbf{u})^2}{2k(T + \Delta T)} = \frac{m(\xi - \mathbf{u})^2}{2kT} (1 + 2(\xi - \mathbf{u})\Delta \mathbf{u} + \dots) \left(1 - \frac{\Delta T}{T}\right) = \\ & \frac{m(\xi - \mathbf{u})^2}{2kT} (1 + 2(\xi - \mathbf{u})\Delta \mathbf{u} + \dots) - \frac{\Delta T}{T} \end{aligned}$$

For large values of the number of particles both formulas are coinciding. In general, we obtain values of various functions. Functionally-Boltzmann equation is invariant with respect to the selection of macro-distribution function. You must compare equilibrium distribution function with macro parameters that taken from the Euler and from the Navier-Stokes equations.

The difference will give us small increments of functions. We obtain that for the Euler equations (the zero approximation of the Chapman-Enskog method) the difference is zero. For the first approximation there are differences. The first approximation is responsible for the tangential component (the viscous stress tensor  $p_{ij}$ ). The Euler equations are obtained using the local-equilibrium distribution function. Consequently, they are responsible for the normal component of the speed, regardless of the values of the macro parameters. When obtaining first order corrections, some of the terms entering the final Chapman-Enskog solution go only after integration with respect to the phase velocity  $\xi$ . The integrals are taken from the function  $f\xi$ , i.e. for  $(\rho u)$ . Consider

$$\begin{aligned} \frac{Df_0}{dt} &= \frac{1}{n} f_0 \frac{\partial n}{\partial t} + \frac{3}{2T} f_0 \frac{\partial T}{\partial t} + \frac{mc^2}{2kT^2} f_0 \frac{\partial T}{\partial t} + f_0 \left(\frac{m}{kT} (\xi - \mathbf{u}) \frac{\partial \mathbf{u}}{\partial t}\right) + \xi \cdot \\ & \left\{ \frac{1}{n} f_0 \frac{\partial n}{\partial x} + \left(-\frac{3}{2}\right) \frac{1}{T} f_0 \frac{\partial T}{\partial x} + \frac{mc^2}{2kT^2} f_0 \frac{\partial T}{\partial x} + f_0 \left(\frac{m}{kT} (\xi - \mathbf{u}) \frac{\partial \mathbf{u}}{\partial x}\right) \right\} = \\ & = 2\mathbf{J}(f_0, f_0 \varphi^k) = \int \mathbf{f}_0 \mathbf{f}_1^0 \left( \varphi_1^{(k)'} + \varphi^{(k)'} - \varphi_1^{(k)} - \varphi^{(k)} \right) \mathbf{g} \mathbf{b} \mathbf{b} \mathbf{d} \mathbf{b} \mathbf{d} \mathbf{e} \mathbf{d} \xi_1 \xi = 0. \end{aligned}$$

In classical case

$$\frac{\partial f_0}{\partial t} \Big|_{t=0} = f_0 \left\{ \frac{m}{kT} \left( c_i c_j - \frac{1}{3} c^2 \delta_{ij} \right) \frac{\partial u_i}{\partial t} + \frac{1}{2T} \frac{\partial T}{\partial t} c_i \left[ \left(\frac{m}{kT}\right) c^2 - 5 \right] \right\}$$

The Boltzmann equation is written relative to the total distribution function and consists of locally-equilibrium function and additional term. The tangential component of the velocity, which is obtained due to the arbitrary direction of the velocity relative to the position of the coordinate axes, is equal to ( -normal )

$\int \mathbf{n} \cdot (\boldsymbol{\tau} \cdot \mathbf{f} \xi) \mathbf{d} \mathbf{s} \mathbf{d} \xi = \int \mathbf{div} (\boldsymbol{\tau} \cdot \mathbf{f} \xi) \mathbf{d} \mathbf{x} \mathbf{d} \xi$  gives us additional term. In addition to locally equilibrium function has a term

$$f_0 \left[ \frac{p_{ij}}{2p} \left(\frac{m}{2T}\right) c_i c_j - \frac{q_i}{p} \left(\frac{m}{kT}\right) \left(1 - \frac{c^2}{5} \frac{m}{kT}\right) c_i \right]$$

The main contribution to the integral will give the derivatives of locally equilibrium distribution function, which determines the self-diffusion equations and thermo-diffusion S.V. Vallander. The second derivative appears due term  $\mathbf{c}_i \cdot \frac{\partial f}{\partial \mathbf{r}_i}$ .

#### IV. CONCLUSION

The paper briefly summarizes the results of previous studies concerning the determination of the influence of the position of the inertia axis of the elementary volume, the lag, the angular momentum for the unstructured particles. The appearance of self-diffusion and thermal diffusion is substantiated. We obtain equations S.V. Vallander across the kinetic results. The presented theoretical results represent the possibility of explaining a number of effects observed in experiments and applications, where role plays plasticity, rapid changes in parameters, mutual penetration of gases and liquids. The result of the impact of new effects may be the emergence of additional force, which is not taken into account in modern engineering calculations

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