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The Application of the Method of Random Search and of Synthesized Algorithm of Dynamic Programming and Random Search to Optimal Designing of Bent Beams

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Abstract − **The paper considers the optimal designing of statically determinate beams of variable stiffness by the random search method with consideration of the restrictions on strength, stiffness and restrictions on the dimensions of the cross sections of the beam under static and dynamic loads. Considered the optimal designing of the console bent beam of variable stiffness using synthesized dynamic programming algorithm and a random search.**

Keywords**-- Optimal Designing, Method of Random Search, Algorithm of Dynamic Programming, Bent Beams**

I. INTRODUCTION

In the design of the elastic structures, the dimensions of which are restricted for reasons of technological, exploitational or other nature, it is often the appears the question about the choice of such structural parameters that would meet the criteria of minimum weight and will satisfy to conditions of strength and stiffness. To solve this rather urgent problem of structural mechanics the apparatus the classical variational calculus is inconvenient, as in a closed area of change of deformation parameters extremum of corresponding functional can not always be achieved on a smooth extremal [1]. Enough promising is the use for the optimal designing of structures of mathematical programming techniques, such as methods of random search. This work should be considered as a message about one of the possibilities of applying a random search to the optimal designing of structures in combination with other methods of mathematical programming.

II. OPTIMAL DESIGNING OF STRUCTURES AT THE PRESENCE OF RESTRICTIONS ON THE STRESSES, STRAINS AND SIZES

The problem of optimal design of elastic systems is seen here in a very general formulation, as the problem of nonlinear programming [2]. For the calculation, is used one of the discrete algorithms of random search. Consider the problem of optimal designing of cantilever beam with rectangular cross-section (Fig. 1), loaded by flat arbitrary system of forces.

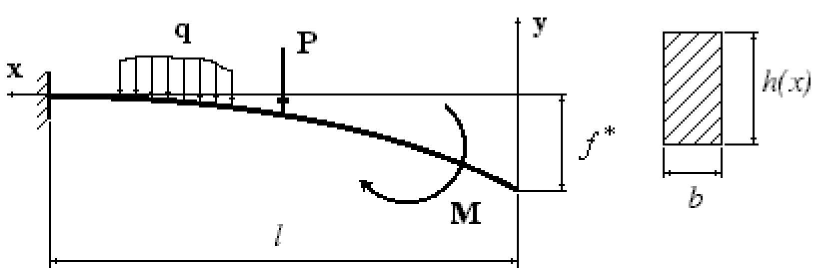

Figure 1. Diagram of a beam with rectangular cross-section, which is subjected to static load

There are the characteristics of the material of cantilever beam: elastic modulus E, specific gravity γ, calculated resistance at bending R, as well as set point allowable sag of the end of the console f^*.

We need to find a such longitudinal profile of beam that at the given loading and a constant width b of the cross-section the beam would have a minimal weight G and would to satisfy the conditions of strength and stiffness, as well as satisfy to geometric constraints on the height of the cross-section of the beam h.

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The problem is reduced to the determination of the minimum of weight function:

$$G = \gamma b \Delta l \sum_{i=1}^{n} h_i \; ; \qquad (1)$$

at the performance of restrictions:

$$c \sum_{i=1}^{n} \frac{k_{1i}}{h_i^3} \le f^* \; ; \qquad (2)$$

$$l\left(1 - C_1 \frac{h_i^2}{x_i}\right) \le 0, \quad (i = 1,2,....,n); \qquad (3)$$

$$h_{\min} \le h_i \le h_{\max} , \qquad (4)$$

Where C and C_1 are defined depending on the nature of the load (for example, at a load of concentrated force P at the end of the console $C = \frac{4Pl^3}{Eb}$, $C_1 = \frac{Rb}{6P}$, and k_{1i} – constant parameters, which depends on the number of sections $\sum_{i=1}^{n} k_{1i} = 1$. Condition (2) is the restriction of stiffness, the condition (3) is the restriction on strength; condition (4) limits the beam's sizes. It is assumed that the ratio between the cross-sectional dimensions are such that the check of the loss of stability the flat form of bending is not needed.

The problem of choosing of the optimal beam parameters (1)-(4) can be formulated as a nonlinear programming problem. The role of the vector of control variables $\mathbf{X}$ in the present case fulfills the vector $\mathbf{H} = (h_1, h_2, ..., h_n)$.

Let us apply for solving the problem (1)-(4) one of the random search algorithms: the proportional algorithm of continuous self-learning with forgetting [3].

To illustrate, consider the problem of finding an optimal beam profile (Fig. 1.), loaded by the force $P = 1{,}9$ kN on the end of the console, for a variety of values f^*, at the following initial data: $l = 1$ m, $b = 0{,}03$ m, $E = 2{,}1 \cdot 10^5$ MPa, $\gamma = 78{,}5$ кN/m^3, $R = 150$ MPa, $0{,}03 \le h_i \le 0{,}1$ m. Beam was divided into 5 sections ($\Delta l = 0{,}2$ m). As a starting point was selected the point with coordinates $h_1 = h_2 = h_3 = h_4 = h_5 = 0{,}1$m, the parameter of forgetting is assumed to be $k = 0.8$; the coefficient of learning $\delta = 20$. Work step in a normalized range of parameters ($\bar{h}_i = \frac{h_i - h_i^{\min}}{\Delta h}$) was equaled $a = 0{,}4$ with the subsequent decreasing till $a/32$. Solutions results are shown in Table 1, and the optimum beam profile is shown in Fig. 2

Table 1
The results of optimal design of the beam, subjected to cross-bending, with the restrictions on stresses, strains and dimensions

№	f^* (cm)	G (N)	Optimal sizes	Cross-sections				
				1	2	3	4	5
1	0,4	138,05	h_i (cm)	3,01	4,83	6,15	7,22	8,09
2	0,5	128,57	h_i (cm)	3,00	4,32	5,59	6,85	7,53

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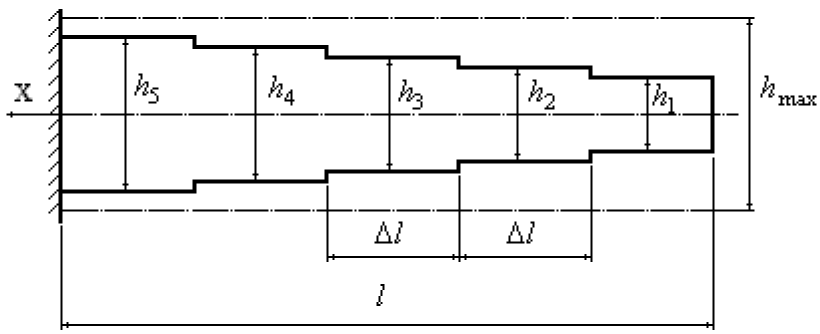

Fig.2. Optimal beam profile

It is interesting to note that the beam is of constant cross section satisfying the restrictions on strength and stiffness (for $f^* = 0{,}4$ cm) at the same load would be have the dimensions $b = 3$ cm, $h = 7$ cm and the weight $G = 165$H, i.e. on 16.4% more than the optimal beam.

III. Optimal Design Of Beams At Forced Vibrations

Consider the optimal designing of elastic structures exposed to dynamic load taking into account the restrictions on strength, stiffness and the restrictions on the geometric dimensions of structures (for reasons of technological or exploitational nature). In the simplified formulation (with no restrictions on the dimensions and stresses), this type of problem is considered in [4, 5, 6] using the apparatus of the classical calculus of variations. For a full account of all the conditions mentioned above variational methods are unacceptable, and therefore there is the need to use modern methods of mathematical programming, oriented on the use of computers.

As in the case of optimal designing of elastic beams at the static load, which is considered above, the problem of optimal design of elastic beam under dynamic loading is put here in the most general formulation, as a problem of nonlinear programming. The method of random search here is used as a mathematical apparatus.

Without the violation the community of statement of the problem, consider a weightless cantilever beam with rectangular cross-section and length (Fig. 1), loaded with the harmonic force $P\sin\theta \cdot t$ ($P = mg$ – its amplitude value, $\theta = \frac{\pi \cdot n_1}{30}$ – the frequency of forced oscillations, n_1 – the number of revolutions of engine per minute).

Are known the characteristics of the material of beam: elastic modulus E, specific gravity γ, calculated resistance at bending R, as well as dynamic set point allowable sag of the end of the console f^*.

$$f_{\max} = y_{\mathrm{st}}\left(1 - \theta^2 m y_1\right)^{-1} \le f^*, \qquad (4)$$

Where the values y_{st} and y_1 are determined, for example, in accordance with [4]. Is required to find a such longitudinal profile of beam that at the given loading P and n_1 a constant width of the cross-section b the beam would have a minimal weight G and would to satisfy the conditions of strength and stiffness (5), as well as satisfy to geometric constraints on the height of the cross-section of the beam $h(x)$. The problem is reduced to the determination of the minimum of weight function:

$$G = \gamma b \Delta l \sum_{i=1}^{n} h_i\ ; \qquad (6)$$

at the performance of restrictions:

$$PA\sum_{i=1}^{n}\frac{k_{i1}}{h_i^3}\left(1 - \frac{\theta^2}{g} PA\sum_{i=1}^{n}\frac{k_{i1}}{h_i^3}\right)^{-1} \le f^* \qquad (7)$$

$$l\left(1 - A_1\frac{h_i^2}{x_i}\right) \le 0, \quad (i = 1,2,\ldots,n); \qquad (8)$$

$$h_{\min} \le h_i \le h_{\max}\ , \qquad (9)$$

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Where: n – the number of plots of beam; k_{1i} – constant parameters, which depends on the number of sections $\sum_{i=1}^{n} k_{1i} = 1$. A and A_1 are defined depending on the nature of the load (for example, at a load of concentrated force P at the end of the console $A = \frac{4Pl^3}{Eb}$, $A_1 = \frac{Rb}{6\mu P}$, and $\mu -$ the dynamic factor). Condition (3.18) is a restriction on the stiffness obtained from (5); the condition (8) is strength condition, and the condition (9) limits the height of the beam. It is assumed that the ratio between the cross-sectional dimensions are such that the check of the loss of stability the flat form of bending is not needed. In general, check the loss of stability the flat form of bending at the optimization of the method of random search does not cause fundamental difficulties.

The problem of selecting the optimal parameters of the beam (6)-(9), thus formulated as a nonlinear programming problem.

The role of the control variables **X** perform the heights of beam in the discrete sections, and the vector of control variables – $\mathbf{H} = (h_1, h_2, ..., h_n)$.

For solving the problem (6)-(9) is used the proportional algorithm of continuous self-learning with forgetting [3]. Consider a numerical example of optimal design of beam (Fig. 3) for different values n_1 and f^* at such initial data: $P = 1{,}9$ кN; $l = 1$ m, b = 0,05m, $E = 2{,}1 \cdot 10^5$ MPa, $\gamma = 78{,}5$ кN/M³, $R = 150$ MPa, $0{,}03 \le h_i \le 0{,}1$ m. The beam was divided lengthwise into 5 sections ($\Delta l = 0{,}2$ m). The results obtained for the optimum profile, and the calculated values of stress in some sections n_1 and f^* them values are listed in Table 3.3 (the case $n_1 = 0$ corresponds to beam of the minimum weight at the static load). Fig. 3 shows a graph of the changing of dynamic factor μ for optimal beam in depending on the number of n_1 revolutions/min. The graph shows that the increase in the value of allowable value of sag leads to increasing of the dynamic coefficient μ.

Table 2
The results obtained for the optimum profiles, and the values of calculated stresses in the cross sections for some values and

n_1 rev/m	$f^\bullet$ (cm)	f_{max} (cm)	Height of cross-section (cm)					G_{min} (kN)	Stresses (MPa)				
			h_1	h_2	h_3	h_4	h_5		σ_1	σ_2	σ_3	σ_4	σ_5
0	0,4	0,400	3,0	4,79	6,.02	7,31	8,18	138,05	89,00	69,5	66,5	60,00	60,00
	1,0	0,995	3,0	3,41	4,35	5,54	5,76	103,94	88,80	137,6	127,6	104,2	120,4
100	0,4	0,400	3,0	4,71	6,15	7,75	8,18	140,38	92,50	75,00	66,50	55,50	62,50
	1,0	0,998	3,0	3,66	4,55	5,48	6,03	107,03	98,80	132,9	126,8	118,4	122,2
200	0,4	0,400	3,09	4,93	6,65	7,65	8,65	145,89	99.00	95,80	64,20	66,00	63,00
	1,0	0,996	3,0	4,00	5,10`	5,92	6,61	116,10	128,6	144,7	133,2	131,7	132,2
600	0,4	0,398	4,76	7,29	9,13	10,0	10,0	193,95	92,00	78,00	75,00	83,20	104,0
	1,0	0,668	4,76	6,68	7,69	9,71	10,0	181,59	148,5	132,6	150,0	125,4	147,9
700	0,5	0,500	6,37	8,80	10,0	10,0	10,0	212,85	74,00	77,30	89,50	120,0	150,0
	1,0	0,500	5,56	9,39	10,0	10,0	10,0	211,76	96,80	67,90	89,90	119,9	150,0

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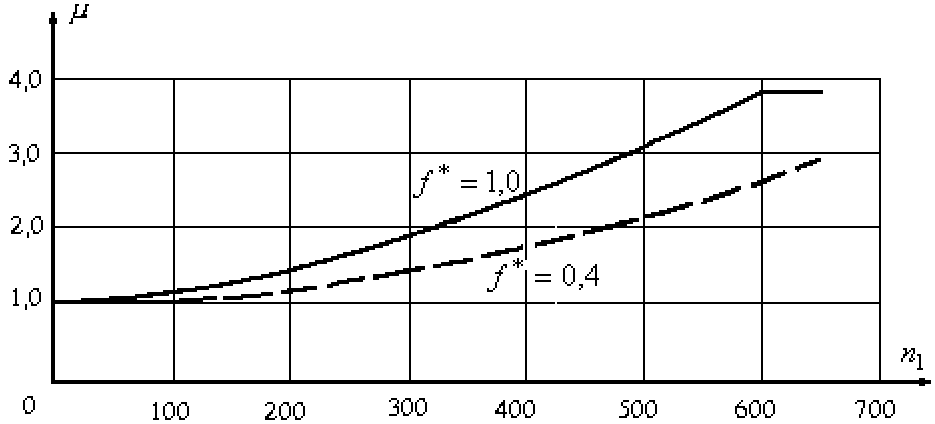

Fig. 3. The dependence of changing of dynamic coefficient for optimal beam on the frequency of forced oscillations for different values of the allowable sag

Analysis of the results listed in Table 2, allows to do concluded that the profiles of optimal beam for relatively large n_1 and f^* are determined by the restrictions on the stresses and on the dimensions of cross-sections of beam. At $n_1 > 700$ rev/min and $f^* = 0{,}01$m the beam completely occupies all gabarite and exhausts the bearing capacity. At this the maximum the sag of the end of console $f = 0{,}005$m has not yet reached the limit value. For small n_1 and f^* the limiting restriction is the value of f^* and, naturally, the beam is not exhausts its load-bearing capacity on strength.

IV. Parameter Optimization Of Bent Beams By The Way Of Synthesized Algorithm Of Dynamic Programming And The Random Search (DPRS)

Synthesized algorithm of dynamic programming, and random search DPRS is described in [7] and is not given here.

For the illustration of using of the algorithm DPRS let us consider the optimal designing of structures made of nonlinear elastic material in the setting of the work [8]. As is known, in many cases, the normal operation of the structure when an elastic-plastic deformation is limited by a permissible value of displacement, the excess of which can lead to structural failure, or to unwanted effects. The problem of optimal designing (on conditions of exploitation) is formulated as follows: it is required out of a set of designs of this type, at reaching simultaneously under the influence of the same load a certain value of displacement in arbitrary predetermined points of the surface, find the parameters of the minimum structural weight (or volume). The corresponding principal stresses, as well as the cross-sectional dimensions should not exceed some predetermined value. When solving this problem we will use the law of deformation in form $\sigma = B|\varepsilon|^{n-1}\varepsilon$ [9]: where B and n the constants of plasticity.

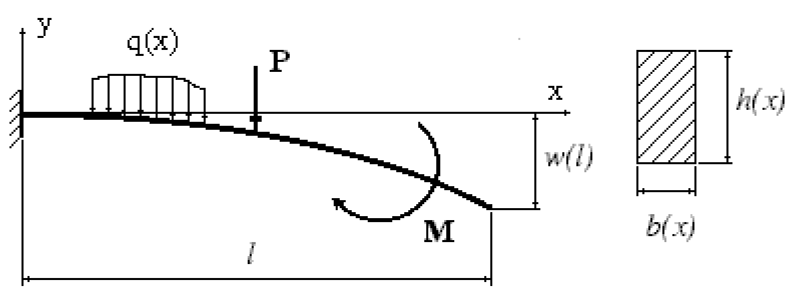

Fig.4. The calculated scheme of a cantilever beam

Let cantilever beam with a rectangular cross-section and length L, of a variable height $h(x)$ and width $b(x)$, is loaded with a lateral load of arbitrary type $q(x)$ (Fig.4)

The differential equation of the elastic line of the beam is of the form:

$$-w'' = B\left[\frac{2n+1}{2n}\right]^n \frac{|M|^n \operatorname{sign} M}{b^n(x)h^{2n+1}(x)}, \quad (10)$$

where x – the axial coordinate; w – sag (vertical deflection); M – the bending moment. It is required to determine such vector of control $\{h(x), b(x)\}$ of dimension $r = 2$, which minimizes the functional:

$$G = 2\gamma \int_0^l \int_0^{h(x)} b(x)h(x)\,dxdz, \quad (11)$$

which is the theoretical weight of the beam, satisfying the equation (10) at all points except for a finite number of points on the interval [0, l]:

$$\eta_p = [w(0), w'(0), w(l), w'(l)], \quad (p = 1,2,....) \quad (12)$$

And satisfying to such conditions that result from the problem statement:

$$b(x) \geq b_0; \quad h(x) \geq h_0; \quad Y_1(x) \leq w(x) \leq Y_2(x); \quad \sigma(x) \leq [\sigma]; \quad \tau(x) \leq [\tau], \quad (13)$$

In the equations (11)-(13) we use the notation: η_p – function limiting conditions; $Y_1(x)$ and $Y_2(x)$ – some specified functions; γ – the specific weight of the material.

For this problem, given that the initial state (for the Cauchy problem) is described by the values $w(0)$ and $w'(0)$, the system of functional equations of dynamic programming has the form:

$$g_N(w_N, w'_N) = \min_{h_N, b_N}\{b_N h_N \Delta + g_{N-1}(w_N, w'_N)\}, \quad g_0 = 0; \quad N = 1,2, \quad (14)$$

where values w_N, w'_N, which are the components of the state vector, is defined by (10) relationship:

$$w_{N-1} = w_N - \Delta w_{N-1}; \quad w'_{N-1} = w'_N + B\left[\frac{2n+1}{2n}\right]^n \frac{|M_{N-1}|^n \operatorname{sign} M_{N-1}}{b^n_{N-1}(x_{N-1})h^{2n+1}_{N-1}(x_{N-1})}\Delta \quad (15)$$

As a numerical example, consider the optimal designing of a cantilever beam the length $l = 5$ m out of the elastic-plastic material. The beam carries a uniformly distributed load intensity of $q = 2$ kN/m at such calculated data $n = 5$, $B = 7{,}6 \cdot 10^{-20}$, and restrictions: $w(l) \leq 0{,}01$ m; $b(x) \geq 0{,}01$ m; $h(x) \geq 0{,}01$ m, $\gamma = 78{,}5$ кН/m^3.

Dynamic programming algorithm (14)-(15) was realized when $N = 10$, and the interval of the state of changing and management was given in accordance with the physical content of the problem in this way: $0 \leq w_N \leq 0{,}01$ m; $0 \leq w'(n) \leq 0{,}005$ rad; $0{,}01 \leq h_N \leq 0{,}3$ m; $0{,}01 \leq b_N \leq 0{,}3$ m.

The number of trials at each stage random search was equal to 30. The number of trials needed to narrow down the search to a point, was equal in limits of 10-17.

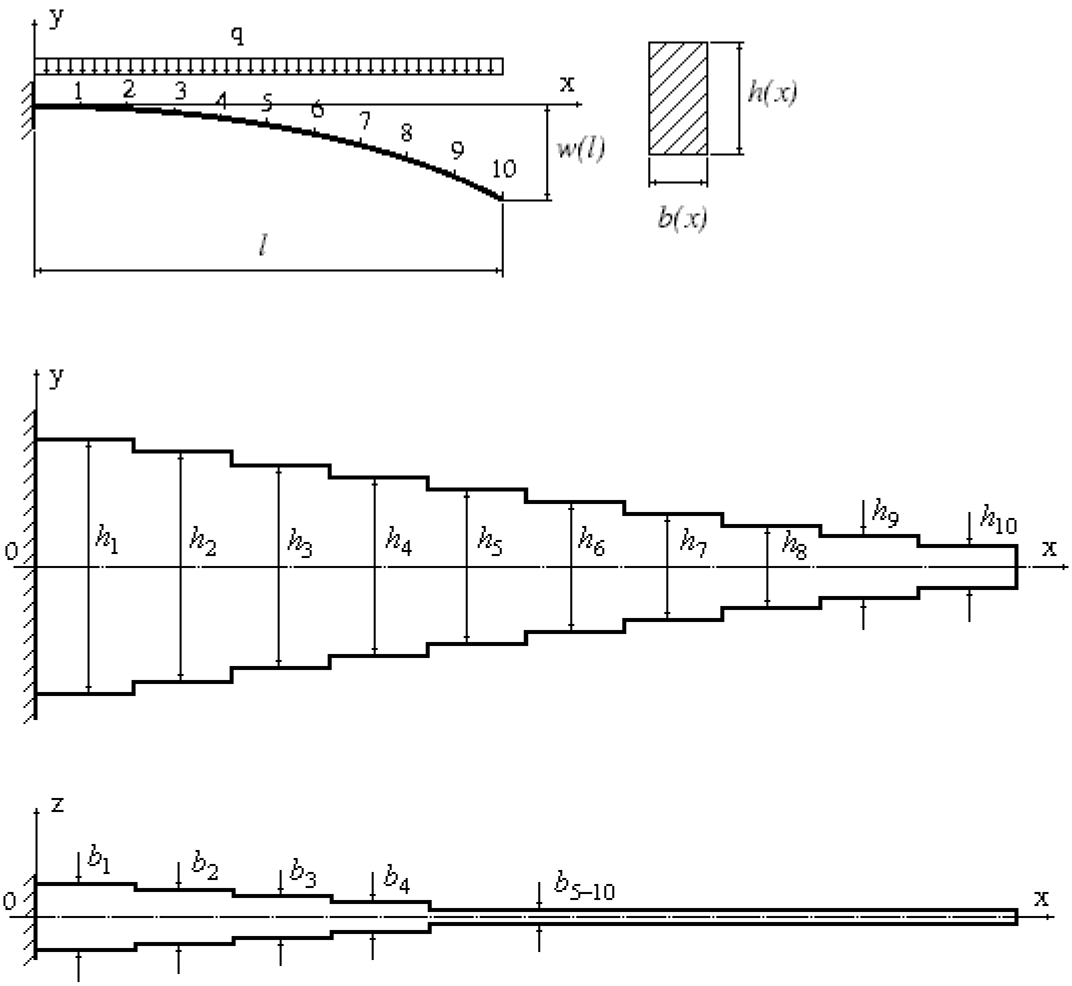

Fig.5. The settlement scheme and an optimal beam profile

Table 3
The results of optimal designing of the beam by the method DPRS

The weight *G*(N)	Dimensions of beam	Numbers of sections									
		1	2	3	4	5	6	7	8	9	10
414,5	*h* (cm)	13,49	12,52	11,67	11,61	10,29	10,17	6,85	4,61	2,83	2,19
	b (cm)	1,590	1,330	1,190	1,150	1,030	1,020	1,01	1,000	1,00	1,00

The obtained optimum sizes of the beam in discrete sections b and h, as well as the value of corresponding minimum weight G are shown in Table 3. The optimum beam profile is shown in Fig. 5.

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