Fast Synchronization Scheme for GSM at Lower SNR

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Abstract-- **The high pace rise in the number of customers for communication systems and associated service provisioning has alarmed the requirement for certain optimal communication technique for optimal service of provisioning. The global system for mobile communication (GSM) has emerged as one of the most potential candidate to for communication systems. There is an inevitable need to enhance the key components of GSM systems such as, modulators and signal decoders. The assurance of these factors can lead low SNR communication in major application scenarios. Furthermore, ensuring efficient synchronization can be of great significance for GSM systems. In this paper, a highly robust and efficient system for low SNR, GMSK synchronization scheme has been developed for GSM system. The proposed joint synchronization scheme encompasses, symbol timing offset estimation, carrier frequency offset and carrier phase offset estimation scheme. To enhance the demodulation, a symbol-by-symbol (SBS) decoder with has been developed. The bit error rate performance analysis affirms that the proposed GMSK synchronization scheme with hard decision based SBS decoder can be a significant development towards enhanced GSM communication system.**

Keywords-- **GSM Synchronization, GMSK Modulation, Symbol by symbol decoder, Low Signal to Noise Ratio.**

I. INTRODUCTION

In the last few decades, the high pace increase in communication system and the well known success of the Global System for Mobile Communication (GSM) has expanded its fin across globe to serve approximate 4 billion users globally[1]. GSM has emerged as a potential communication technology across industrialized as well as non-industrialized countries and it has established itself as a reliable solution for sophisticated 3rd generation and 4th generation communication systems. Interestingly, in numerous countries GSM is the only cellular network accessible till. The recently introduced enhanced GPRS (EGRPS) system, GSM can facilitate data rates of up to 1.2 Mb/s by means of enhanced modulation approached like 16 QAM and 32 QAM, and effective synchronization schemes etc. in general, the data retrieval of higher order modulation EGPRS signals at certain adequate SNR levels needs higher end radio frequency (RF) transceivers having very low noise figure[2] with effective channel equalization and respective demodulation mechanism at receivers[3].

To facilitate the low cost characteristic of 2G devices, the sophisticated and low complexity solutions are needed that might ensure efficient communication even at low SNR-levels and this is a key issue in the digital baseband of GSM based receivers. The efficiency of Gaussian Mean Shift keying (GMSK) based modulation technique has enhanced the GSM performance dramatically that enables it to deliver reliable solution even at lower SNR conditions with the input signal power level of -110 dBm that represents an SNR of approximate 7 dB.

This is the fact that in communication system, numerous modulation approaches depicts varied functional tradeoffs between the noise tolerances and cost in addition to other interferences as well as spectral efficiency etc. GMSK is one of the most employed modulation technique which is a type of the MSK modulation approach. In GMSK modulation technique, the phase of the carrier signal is varied constantly using a Gaussian filter shaped antipodal signal. Being a member of MSK modulation technique family, GMSK possesses the modulation index of 0.5. The implementation of the Gaussian filter emphasizes the energy, thus permitting lower band power output. The unvarying envelope permits GMSK modulation technique to be comparatively less vulnerable to the fading channel and hence an ideal mechanism for GSM communication. The symbol-by-symbol (SBS) demodulator[4] is considered to be a robust candidate for GMSK because of its robustness towards efficient decoding and less complex architecture as compared to Viterbi Algorithm (VA). To design an efficient demodulator, there is an inevitable need for perfect synchronization[4]. This paper intends to explore an efficient synchronization approach for GMSK modulation in GSM system.

Synchronization states a multi-parameter estimation issue that comprises the synchronization of key parameters such as symbol timing offset, carrier frequency offset and carrier phase offset. A number of researches have been done so far to enhance synchronization. In general, the approaches like maximum-likelihood or maximum-a-posteriori based joint estimation are of theoretical significance but are typically intricate in implementation[5]. Taking into consideration of the operational complexity and robust environment requirement in GSM communication system, fast synchronization algorithms permit low complexity.

In this paper, we have emphasized on feedforward synchronization paradigm to alleviate the issues of hang-up problem that is common in major feedback based approaches[6]. An effort was made to implement complete feedforward scheme for joint carrier offset estimation and symbol timing offset estimation for MSK signals[6]. Unfortunately, it can't implement narrow-band GMSK signals due to poor performance[7]. An approach for joint frequency and timing recovery was suggested using MSK modulation technique that incorporated an estimation approach by combining multiple correlation functions having varied time lags[5]. Unfortunately, this approach turns out to be much complicated for GSM implementation. One more effort was made where a combined time and phase synchronization approach[8] was proposed that was employed with MSK signals. Then while, the influence of the carrier frequency offset could not be examined. A well calibrated synchronization scheme for GMSK can facilitate optimal performance and can be a flexible alternative for digital implementation[9]. The existing systems for synchronization in GSM mobile communication are not sufficient while considering noise, power and mobility constraints. In this paper, a low SNR GMSK synchronization scheme has been proposed for GSM communication system. A brief of the implemented GSM system is given in the following section.

II. GSM Transceiver: A Glance Of System Model

This section discusses the implementation of GSM system and its synchronization. An overview of the proposed GSM model is presented in Figure 1.
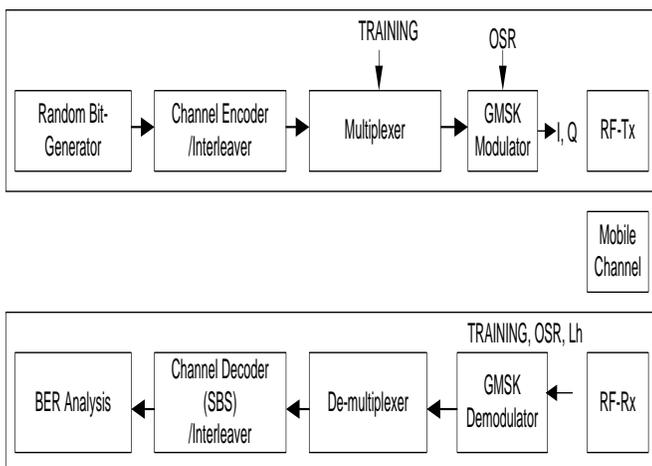

Figure 1. Conceptual block diagram for a GSM transceiver

In this paper, we have used burst mode transmission for GSM simulation and a single GSM data block with 260 random bits has been generated. The generated random data bits have been feed as input data stream to the channel encoder and then interleaver. The interleaved data are then processed by a multiplexer (MUX) that splits the incoming sequence to form a GSM normal burst. Since, the burst type of input data requires certain training data sequence. In this model a 26 bit training signal has been introduced. Generating the GSM burst data the multiplexer returns it to the GMSK modulator that performs differential encoding of the incoming burst to form a Non Return to Zero (NRZ) sequence, which is then subjected for GMSK modulation. The resulting signal is presented in terms of a complex baseband signal comprising real and imaginary signals, I and Q respectivelly. In our simulation model, the number of samples per data bit has been defined in terms of the oversampling rate (OSR). In our developed model OSR of value four (OSR=4) has been used. Considering realistic simulation environment, in our simulation Gaussian and Raleigh multipath fading channel has been used for simulation.

On the receiver side of the developed GSM model, three operational blocks have been developed. These are demodulator, demultiplexers, and channel decoder. As illustrated in Figure 1, the GMSK demodulator receives a GSM received burst by means of a complex baseband representation. On the basis of this data sequence, OSR, training sequence, and the expected length of the receiving filter L_h, the GMSK demodulator estimates the most probable bit sequence. Hence, the demodulated sequence is then employed as input to the demultiplexer where the bits are processed for splitting to recover the real data bits from the sequence. At this stage the other signal bits such as the control bits and the training sequence are released. Performing demultiplexing, the channel decoder has been done. In this paper the signal by signal (SBS) decoder has been implemented on the reconstructed sequence delivered by the channel decoder. This paper focuses on a joint synchronization paradigm for GSM system and has implemented SBS MAP decoder for data retrieval. The following sections focus on the implemented modulation, synchronization and decoding technique. The overall BER analysis has exhibited that the proposed joint synchronization approach enhances the performance of GSM system.

III. GMSK Modulator

Gaussian Minimum Shift Keying (GMSK) algorithm has been the most suitable approach of the continuous phase modulation (CPM) mechanism.

The higher bandwidth efficiency and constant envelope modulation feature strengthens GMSK modulation for GSM systems (B=0.3) in mobile communication systems. Since last few decades CPM scheme has been explored to enable higher communication efficiency, better spectrum utilization and power efficiency. Primarily, CPM schemes are categorized into two broad types, called full response and partial response on the basis of the fact whether the modulation frequency pulse is of single symbol duration of longer. MSK is one of the popular types of a full response spectrally efficient modulation scheme. On the other hand, GMSK scheme is the most generic type of modulation scheme due to higher spectral efficiency and constant envelope modulation characteristics. GMSK modulation scheme has h=0.5 partial response CPM scheme originated from MSK with the addition of baseband Gaussian filtering implemented to the identically and distinctly distributed random rectangular pulse shaped input signal earlier to the frequency modulation of the carrier signal. The following section discusses the GSM receiver architecture and the proposed synchronization scheme at the GSM receiver.

IV. GSM RECEIVER SIGNAL MODEL

In our implemented GSM model, the complex envelope of the received baseband GMSK signal is obtained as:

$$r(t) = e^{[2\pi vt + \theta]} s(t - \tau) + w(t) \qquad (1)$$

where v represents the carrier frequency offset, θ represents the carrier phase offset and τ states for symbol time offset. The noise component $w(t)$ states the complex valued Gaussian and Rayleigh fading channel noise with real and imaginary signal components, individually possessing two-sided power spectral density given by $\sigma^2 = N_0/2E_b$ where E_b represents the received signal energy per symbol. The transmitted signal $s(t)$ is given by

$$s(t) = e^{j\psi(t;a)} \qquad (2)$$

Where $\psi(t; a = \pi \sum_k a_k q(t - kT))$ represents the information bearing phase. Here, $z = a_i$ refer the data symbols having the values of ± 1 with equal likelihood. The variable T represents the symbol period while $q(t)$ states the phase pulse of the modulator. It is in fact, the integration of the frequency pulse $g(t)$. In order to facilitate the estimation of symbol time offset, carrier frequency offset and carrier phase offset, $s(t)$ in the nth duration $(nT \le t \le (n+1)T)$ can be represented as follows[10].

$$e^{j\psi(t;a)} = exp\left(j\frac{\pi}{2}\sum_{k=0}^{n-2} a_k\right) \prod_{k=n=1}^{n} exp[j\pi a_k q(t - kT)] \qquad (3)$$

$$\alpha_{0,n} = exp\left(j\frac{\pi}{2}\sum_{k=0}^{n-2} a_k\right) \qquad (4)$$

$$\text{and} \alpha_{0,n} = (ja_n)\alpha_{0,n-2}.$$

Considering (2), it can be found that the baseband signal $s(t)$ can be represented in a linear form[10] as depicted below:

$$s(t) = e^{\psi(t;a)} = \sum_{i=0}^{n} \sum_{k=n=2}^{n} a_{i,k}\, h_i(t - kT) \qquad (5)$$

The probable values of the parameter $\alpha_{1,n}$ can be $(ja_n)\alpha_{0,n-2}$. In our developed model, we have initialized $\alpha_{0,-2} = 1$, without any loss of generality by considering that there is no data transmission to time $t = -T$ and $\alpha_{-1} = 1$. In our proposed model, the nonlinear GMSK signal $s(t)$ has been decomposed into sums of amplitude modulated (AM) pulses in two dimensions and the two pulse shaping filters $h_0(t)$ and $h_1(t)$ have been employed. The impulse shaping filter has been obtained as[10] $h_0(t) = p(t - T)p(t - 2T)$ for $t \in [0,3T]$ and $h_1(t) = p(t - 2T)p(t + T)$ for $t \in [0,T]$, where the variable $p(t)$ refers

$$p(t) = \begin{cases} \cos(\pi q(t), & t \in [0,2T] \\ p(-t), & t \in (-2T,0) \\ 0, & |t| \ge 2T. \end{cases} \qquad (6)$$

In case of the GMSK signals, $g(t)$, which is the frequency pulse represents the convoluted output of a low-pass Gaussian filter having a rectangular pulse in the duration of T and magnitude $1/(2T)$. Mathematically, it is represented as follows:

$$g(t) = \frac{1}{2T}\left\{Q\left[\frac{2\pi B}{\sqrt{ln2}}\left(t - \frac{3T}{2}\right)\right] - Q\left[\frac{2\pi B}{\sqrt{ln2}}\left(t - \frac{T}{2}\right)\right]\right\} \qquad (7)$$

In (7), B represents the 3 dB bandwidth of the implemented Gaussian low-pass filter and $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}\, dt$. In general, $g(t)$ is time truncated in the interval of $[0, LT]$, which is further normalized as $\int_0^{LT} g(t)dt = \frac{1}{2}$.

In GSM system, the design parameter BT has been selected as 0.3 and L=2, as $g(t)$ is found to be approximate zero in case of $t \geq 2T$. Now, substituting $s(t)$ by means of its linear equation as presented in (5), the signal in its discrete form can be represented as follows:

$$_{,n} = e^{j\left[2\pi v\left(kT + n\frac{T}{N}\right) + \theta\right]} \qquad (8)$$
$$\times \sum_{i=0}^{1} \sum_{l=k-2}^{k} \alpha_{i,l} h_k\left((k-l)T + n\frac{T}{N} - \tau\right) + w_{n,i}$$

Here, the sampler of the received signal is denoted by $r_{k,n}$ while, $w_{n,i}$ represents the noise at time $t = kT + n\frac{T}{N}$, where N signifies the OSR value and $T_s = T$ sampling time. In fact, the linear presentation of the GMSK scheme is equivalent to the PAM modulation except with the inclusion of inter-symbol interference (ISI). In the case of no ISI, the signal may be reduced to the MSK signal. These features motivate us to develop certain enhanced synchronization scheme for GMSK by amalgamating these approaches for MSK so as to reduce noise of interference to a significant level. In this paper, we have proposed a joint synchronization paradigm to enhance BER performance. A brief of the proposed synchronization schemes for symbol timing, phase and frequency is discussed in the following sections.

A. Symbol Timing Offset Estimation

The MCM scheme is particularly developed for MSK scheme and in case of its implementation with GMSK; it depicts degraded results[11]. Then while, it has been found that the feedforward mechanism based timing recovery can be highly efficient in case of burst transmission. Since, in this paper, we have developed a burst transmission based GSM system, the feedforward mechanism can be of considerable significance. Here, we have introduced a modified time offset estimation algorithm to enhance GSM performance in mobile communication environment. The predominant concept behind the implementation of MCM scheme is to implement the nonlinear combinations of the delayed versions of the baseband signal comprising certain periodic signal components, which may be easily exploited for clock recovery. In general, to perform synchronization of the MSK signals, the following fourth-order nonlinear transform is employed.

$$z(t) = E\{[x(t)x^*(t - mT)]^2\} \qquad (9)$$

where $E\{.\}$ represents the anticipation function, while the variable m refers an integer. In case $x(t) = r(t)$, the variable $z(t)$ can be represented as

$$z(t) = e^{j4\pi m v T} g(t - \tau) + n(t) \qquad (10)$$

Where $n(t)$ refers noise and $g(t)$ refers the periodic signal. Thus, the periodic signal can be presented by

$$g(t) = E\{e^{j2[\psi(t;a)-\psi(t-mT;a)]}\} \qquad (11)$$
$$= \prod_{k=-\infty}^{\infty} cos(2\pi[q(t-kT) - q(t-(k+m)T)]) \qquad (12)$$

Thus, considering equation (10), the timing information has been obtained.

For MSK signals, $m = 1$ is used as per suggestions for MCM scheme[11]. In order to enhance the performance of the proposed GMSK modulation technique, a number of periodic signals having distinct m have been combined to calculate the time offset[12], still this scheme suffers from higher computational complexity. Hence, in this paper, we have employed a simplified correlation function having $m = 2$. Our proposed scheme is equivalent to the MCM scheme which is applauded due to its flexible implementation with hardware. In our proposed approach, $r(t)$ has been filtered using a low-pass filter that results into enhanced SNR for time offset calculation. Here we have used the matched filter to enhance the SNR at the GSM receiver. In our proposed system, taking into consideration of the fact that $h_1(t) \ll h_0(t)$, only one-dimensional matched filter[10] $h_0(t)$ has been used that significantly reduces complexity. The result of the matched filter has been fed into the time offset estimator and hence the prime difference of our proposed system and others[11, 12] is the input to the nonlinear transform function. In the proposed system the $x(t)$ is obtained by

$$x(t) = r(t) \otimes h_0(-t) \qquad (13)$$

Consider $x_{k,n} = x(t)|t = kT + nT_s$. Thus, the overall timing synchronization can be obtained by

$$= -\frac{T}{2\pi} arg \sum_{n=0}^{N-1} \left\{ \left[\sum_{k=0}^{L_T-1} [x_{k,n} x^*_{k-2,n}]^2 \right] e^{-j2\pi n/N} \right\} \qquad (14)$$

where $arg(.)$ represents the phase processing, and LT represents the observation period for timing synchronization.

B. Carrier Frequency Offset Estimation

Performing symbol timing offset estimation; we have interpolated the received signals to get the sampler at the correct sampling time and the received signal which is already time synchronized is employed for carrier frequency offset estimation. In this paper, we have used preamble added or data added carrier frequency offset estimation scheme for synchronization. A well known frequency estimator is Fitz scheme[13] that employs the phase of the correlation associated with the delayed versions of the demodulated signals, then while its performance primarily depends on the delay. In order to enhance the estimation accuracy, a large delay is anticipated that might confine the synchronization range. On contrary, in GSM systems, the synchronization range is expected to be wider because of robust functional dynamicity and hardware characteristics. To reduce the delay, a maximum delay cap is introduced, which is obtained by

$$D_{max} < \frac{1}{2|v_{max}|T} \qquad (15)$$

Because of the shortcomings in D_{max}, the preciseness of the carrier frequency offset might be compromised in case of Fitz algorithm implementation. In this paper, the generic Fitz scheme has been enhanced to the sample level and thus the ultimate sampling of the demodulated signal has been obtained as follows

$$y(iT_s) = r(iT_s)s^*(iT_s - \hat{\tau}) \qquad (16)$$
$$= e^{j[2\pi v(kT+nT_s)+\theta(iT_s)]} + n'(iT_s)$$

Where $k = \left\lfloor \frac{i}{N} \right\rfloor$ and $n = i - Nk$. Here, $\phi(iT_s)$ result from the timing estimation error and other variable $n'(iT_s)$ represents the zero-mean noise. In case of accurate time estimation, the impact of $\phi(iT_s)$ has been neglected and thus the carrier frequency offset, $\hat{v}$ has been obtained by

$$\hat{v} = \frac{1}{\pi D(ND+1)T} \sum_{m=1}^{ND} argR(m) \qquad (17)$$

where D represents a parameter less than D_{max} and $R(m)$ represents

$$R(m) = \frac{1}{NL_{f-m}} \sum_{i=m}^{NL_{f-m}} y\big((iT_s)y^*(i-m)T_s\big) \qquad (18)$$

Where L_f represents the observation period for carrier frequency offset estimation.

Our proposed scheme functions at the sample level that might enhance the overall performance and it is because of the increased delay length at the sample level due to multiplication with the oversampling rate (OSR) N.

C. Carrier Phase Offset Estimation

The carrier frequency offset introduces certain phase rotation in $y(iT_s)$ (16) that can be significantly alleviated by employing $\hat{v}$ in (17). Performing carrier frequency offset alleviation; the carrier phase offset estimation can be performed by

$$\hat{\theta} = tan^{-1} \frac{\sum_{i=0}^{L_\theta - 1} I[y_c(iT_s)]}{\sum_{i=0}^{L_\theta - 1} R[y_c(iT_s)]} \qquad (19)$$

where $R[.]$ and $I[.]$ are the real and imaginary part of a complex signal, respectively. Here, L_θ represents the duration for the phase synchronization. Finally, $y_c(iT_s)$ represents the compensated (carrier frequency offset) and demodulated signal. Mathematically, it can be expressed as

$$y_c(iT_s) = e^{j[2\pi(kT+nT_s)\hat{v}]}y(iT_s) \qquad (20)$$

where $k = \left\lfloor \frac{i}{N} \right\rfloor$ and $(n = i - Nk)$.

Here, it must be noted that L_θ varies as per variation in the phase property and the higher L_θ assures better phase estimation, under the condition that θ is constant during L_θ. Still, it can be found that because of the residue carrier frequency offset introduced by certain imperfect frequency estimation, the phase gradually but steadily changes over time and therefore L_θ cannot be more than the logical phase processing time. In this paper, introducing preamble, the initial value of θ has been estimated which has been updated during the data transmission. To decode the data, in our proposed model, we have employed symbol by symbol (SBS) MAP decoding scheme. A brief of the implemented SBS MAP algorithm is given in the following section.

V. SYMBOL BY SYMBOL DEMODULATION

This is the matter of fact that the symbol-by-symbol (SBS) maximum *a posteriori* probability (MAP) algorithm, also known as SBS MAP algorithm is an optimum decoding algorithm for codes , which can be presented by a trellis of finite duration[14-17]. In this paper, SBS MAP demodulation has been employed, which has been derived for the case of continuous phase modulation (CPM) signals transmitted over Gaussian and Rayleigh flat-fading channels, and a corresponding receiver structure, as already discussed in above sections.

The proposed SBS MAP algorithm needs estimating the sum of the products (SOP) of the weights of all traces or paths across the trellis which pass through that specific branch. Such computation can be significantly enhanced by means of certain forward and backward recursion scheme across the trellis. In this paper, we have explored the strengths of an existing literature[18] and we have used MAP algorithm for demodulation in our proposed GSM systems. SBS MAP algorithm employs *a priori* symbol probabilities function at its input and generates optimal decisions as its output. Since, this approach is well suited for iterative process based applications, where processed and refined input symbol probabilities are iteratively fed back to the demodulator as *a priori* information. Thus, the ultimate refined results and respective decisions generate enhancements in the successive phases. SBS MAP algorithm has illustrated significant performance for decoding utilities for communication systems[19,20,21]. A brief of its functional approach is given as follows:

Consider certain received sample sequence, the predominant objective of SBS MAP demodulation algorithm is o estimate all feasible symbols, respective times, and the likelihood that certain symbol was transmitted at that specific time instant. Determining the probabilities of these parameters, the demodulator may than employ them to perform decisions and further they can be employed for extracting bit soft decisions. In this section, the process for demodulating certain CPM signals with the conditional probability density values on the Gaussian and Rayleigh flat-fading channels has been discussed. Some of the variables used for SBS demodulator are given as follows:

$B\ B \rightarrow$ *Transition Probabilities of the states*
$Q \rightarrow$ *Number of data symbols*
$N \rightarrow$ *The length of data block, in symbol periods;*
$M \rightarrow$ *number of states in trellis at a given time;*
$K \rightarrow$ *hypothesis;*
$R \rightarrow$ *number of samples/symbol*
$T \rightarrow$ *discrete-time index, in symbol periods;*
$U \rightarrow$ *transmitted symbols;*
$X \rightarrow$ *transmitted symbol samples;*
$Y \rightarrow$ *received symbol samples;*
$U(k) \rightarrow$ *Q-ary input symbols;*
$X(k) \rightarrow$ *Transmitted samples.*

In our developed SBS modulator, it has been assumed that the collective memory of the channel and the modulator is lower than the K symbol periods so that any received sample is impacted by not more than $K + 1$ successive input symbols. Hence, as a result the trellis is formed less than $M = Q^k$ states can be used.

Here, the individual trellis node posses Q input branches and same output branches, where the individual branch corresponds to one of the Q data symbols. Here, it can also be considered that in our proposed GSM system, our proposed system functions on a block of data which starts and ends in certain acknowledged state. An approach to obtain such condition may be to start and end the individual data block with certain known symbols. B, the state transition probabilities is of great significance because these can be employed for estimating the probability that a symbol was transmitted at certain time t. Now, consider the subset of the set of hypotheses $\{k\}$ be $C_t(m',m)$ which traverse certain trellis branch in between the states $s_{t-1} = m'$ and $s_t = m$. In our proposed model, the state transition probability has been estimated by

$$P_r\{ s_{t-1} = m'; s_t = mY\} \qquad (21)$$
$$= \frac{\sum_{k \in C_t(m',m)} \Pr\{\dot{U} = U(k)Y\}}{\sum_k \Pr\{\dot{U} = U(k)Y\}}$$

In order to estimate the state transition probability (21), the parameter $|\Pr\{\dot{U} = U(k)Y\}|$ has been obtained using Bayes' theorem[22], which is expressed as

$$\Pr\{\dot{U} = U(k)Y\} = \frac{p(Y\dot{U} = U(k)) . \Pr\{\dot{U} = U(k)\}}{p(Y)} \qquad (22)$$

Substituting (22) in (21), it can be noticed that all the terms would have similar denominator that would be cancelled and hence can be ignored in calculation. To evaluate the numerator in (22), in our model, we have assumed that the $Q - ary$ input symbols, which are transmitted at certain distinctive time instants, are usually independent. Hence, the second factor of the numerator in (22) is turns out to be

$$\Pr\{\dot{U} = U(k)Y\} = \prod_{t=0}^{N-1} \Pr\{\tilde{u}_t = u_t(k)\} \qquad (23)$$

Now, considering first factor in the numerator of (22), $p(Y\dot{U} = U(k))$, it can be found that there exists a 1-to-1 connection between the input vectors $U(k)$ and the transmitted sample vector $X(k)$. Hence, as a result, it is obtained as

$$p(Y\dot{U} = U(k)) = p(Y\dot{X} = X(k)) \qquad (24)$$

Thus, the right-hand side of (24) is obtained by

$$p_{y|x}[Y|X(k)] = p_{y|x}\left[(y_{rN-1}, \dots, y_1, y_0)\big|X(k)\right]$$
$$= \prod_{j=0}^{rN-1} p_{yj}\, y_{j-1}[y_j|y_{j-1}(k)] \qquad (25)$$

Where

$$y_j(k) = \{(y_j, \ldots, y_1, y_0), X(k)\} \qquad (26)$$

Here, the estimation of the factor $p_{y_j}|\,y_{j-1}[y_j|y_{j-1}(k)]$ depends on the channel model and modulation scheme, which in our proposed system are Gaussian and Rayleigh fading channel and GMSK modulator respectively. Now, here we consider that the computation does rely primarily on $K+1$ symbols associated with a trellis possessing Q^k states. During the formation of trellis and for any trellis branch, the individual feasible symbol at certain specified time is exclusively characterized by the starting and ending states of the subsequent branch at that time. Hence, in our proposed SBS algorithm there is a 1-to-1 connection between the input symbols sequence $U(k) = \{u_1(k) \cdots u_{N-1}(k)\}$ and the state transition sequence $M(k) = \{(m', m)_0(m', m)_1 \cdots (m', m)_{N-1}\}$, where $\{(m', m)_t\}$ states the initial and end (i.e., starting and ending) states for kth hypothesis at certain time t. In our developed algorithm m' at time t is equivalent to that of m at time $(t-1)$.

Consider,

$$W_t(m', m) = \prod_{j=rt}^{r(t+1)-1} p_{y_j}\,y_{j-1}[y_j|y_{j-1}(k)] \qquad (27)$$

Now, substituting equation (27) into (25) gives

$$p_y[Y\,X(k)] = \prod_{t=0}^{N-1} W_t(m', m) \qquad (28)$$

Where $\{W_t((m', m)\}$ is independent of the hypothesis k as the estimation of $p_{y_j}y_{j-1}[y_j y_{j-1}(k)]$ depends only on $K+1$ successive symbols. Now taking into consideration of the equations, (23, 24, 28), the numerator of (22) becomes

$$
\begin{aligned}
&p\left(Y\tilde{U} = U(k)\right).Pr(\tilde{U} = U(k))\\
&= \prod_{t-1}^{N-1} Pr\{\tilde{u}_t = u_t(k)\}.W_t(m', m) \qquad (29)\\
&== \prod_{t-1}^{N-1} \daleth_t(m', m)
\end{aligned}
$$

Where $\daleth_t(m', m) = Pr\{\tilde{u}_t = u_t(k)\}.W_t(m', m)$. As the sequence of multiplicative branch weights is a function of the hypothesis k, individual multiplicative branch weight $\daleth_t(m', m)$ are not functions of the hypothesis k due to the reason that the individual branch is connected to certain specific symbol and hence all those hypotheses which pass through certain specific branch encompasses the specific symbol in that duration. Thus, finally, the state transition probability, as defined in equation (21) becomes

$$Pr\{S_{t-1} = m'; S_t = mY\} \qquad (30)$$
$$= \frac{\sum_{k \in C_i(m'_t, m_t)} \prod_{i=0}^{N-1} \daleth_i(M_i(k))}{\sum_k \prod_{i=0}^{N-1} \daleth_i(M_i(k))}$$

Where $M_i(k)$ represents the i th element of the hypothesized path through the trellis $M(k)$. For the purpose of signal demodulation in GSM (GSM Standard GSM 05.03), we are interested in estimating $Pr\{u_t = qY\}$ where q refers one of the Q-ary input symbols, which is input to the SBS MAP demodulator. In our proposed decoder, it has been estimated by adding the overall transition probabilities corresponding to those all branches allied with the symbol q at certain time t. In this paper, computations based on soft as well as hard decisions based demodulation has been done, where decoding using hard decision has demonstrated better performance as compared to soft decision process. The final decoded signals have been used for bit error ratio analysis. The results obtained for BER analysis with different SNR and Eb/N0 are discussed in the following section.

VI. RESULTS AND DISCUSSION

In this paper, a novel synchronization scheme for Gaussian Minimum Shift Keying (GMSK) algorithm has been developed for low SNR, GSM system. To simulate GSM system, a burst mode transmission scheme has been developed for GSM simulation, where a single GSM data block with 260 random bits has been employed as input data sample. The overall simulation model has been developed using MATLAB software and GSM 05.05 (3GPP TS 05.05 standard) standard. The overall developed transmitter and receiver components comprises data generator, differential coding, interleaver, multiplexer, GMSK modulator and GMSK demodulator, demultiplexer, de-interleaver, SBS based decoder, respectively.

Here for simulation, BT=0.3 (GSM standard), oversampling rate (OSR) of 4 and sample time $T = 3.692 \times 10^{-6}$ has been considered. In this paper we have proposed a joint synchronization scheme that intends to synchronize symbol time offset, carrier frequency offset and carrier phase offset altogether. Unlike other GSM synchronization schemes, in this paper we have used three variables (time, frequency and phase) based synchronization altogether. In this paper, we have developed a symbol by symbol (SBS) maximum *a posteriori* probability (MAP) algorithm, named SBS-MAP algorithm for signal decoding, which has been followed by Bit Error Ratio (BER) analysis. Figure 2 represents the BER performance of our proposed GMSK modulation based synchronization scheme for GSM system. Figure 3 depicts signal to noise per bit (Eb/No) for the developed system. GSM 05.05 and other standards for GSM communication system suggests low SNR up to 7dB for mobile communication. These hard decision based SBS decoding affirms better performance. Following figure affirms the standard requirements (Figure 2 and Figure 3) for GSM systems. Figure 4 represents the BER performance with varying block size or data frame, where the results obtained assures better performance with decreasing SNR slope as per increase in number of data blocks.
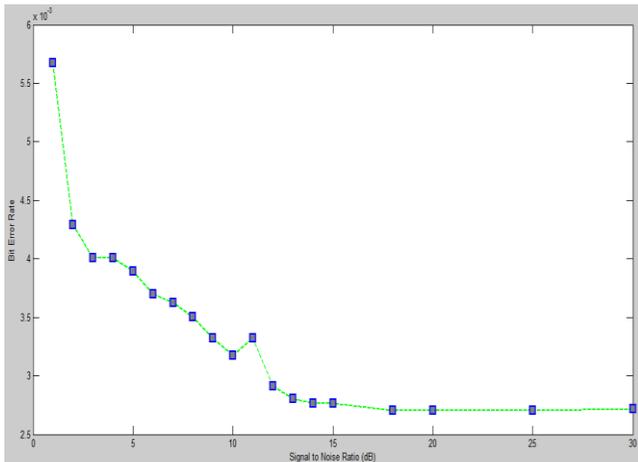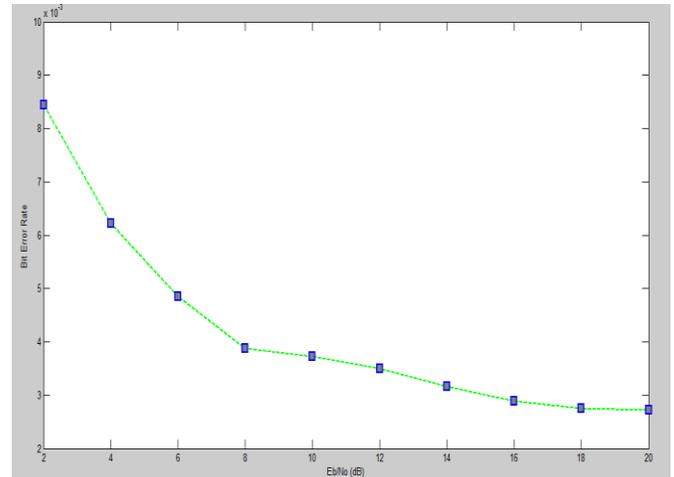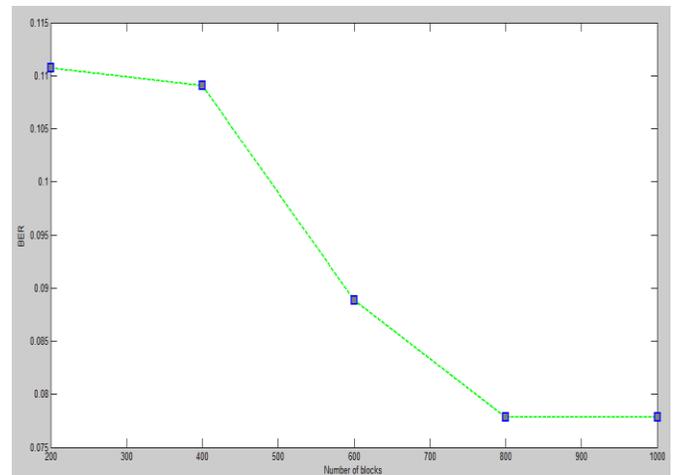

Figure 3. BER Vs Eb/No

Figure 4. BER Vs Number of Blocks (Frames)

A reference work[23] has developed a joint synchronization scheme for GMSK, which has been employed for GSM system. Researchers have used Viterbi-adaptive equalization and soft decision Viterbi decoding approach. Table 1 represents the comparative BER performance of the proposed synchronization scheme. Recalling our proposed scheme for symbol timing, frequency and phase synchronization scheme for GMSK modulation and SBS based decoding, here it can be found that our proposed system perform better as compared to existing scheme.

Figure 2. BER Vs SNR

Table 1
BER performance

SNR	Bit Error Rate	
	Existing[23]	Proposed
0	0.3032	0.0071
1	0.2664	0.0064
2	0.2263	0.0062
3	0.1843	0.0054
4	0.1424	0.0047
5	0.1028	0.0042
6	0.0683	0.0042
7	0.0407	0.0041
8	0.0213	0.0036
9	0.0094	0.0035

Table 2
BER performance for varied decision decoders

SNR	BER for Decision Decoder	
	Soft Decision	Hard Decision
0	0.3527	0.0071
1	0.3525	0.0064
2	0.3522	0.0062
3	0.3521	0.0054
4	0.3520	0.0047
5	0.3520	0.0042
6	0.3519	0.0042
7	0.3519	0.0041
8	0.3517	0.0036
9	0.3517	0.0035

Figure 5 represents the comparative BER performance of the proposed and existing system[23]. Considering VA as a conventional candidate for decoder researchers[23] have suggested soft decision based decoding. Interestingly, in the proposed SBS decoder based decoding, the hard decision decoder has exhibited better results (Table 2). In Figure 6, where green dotted line states hard decision decoder based BER performance while the soft decision decoder based performance is illustrated in red dotted line. Here, it must be noted that considering real time application environment, the proposed system considers both the Gaussian noise channel as well as Rayleigh fading channel that assess realistic performance of the proposed scheme.
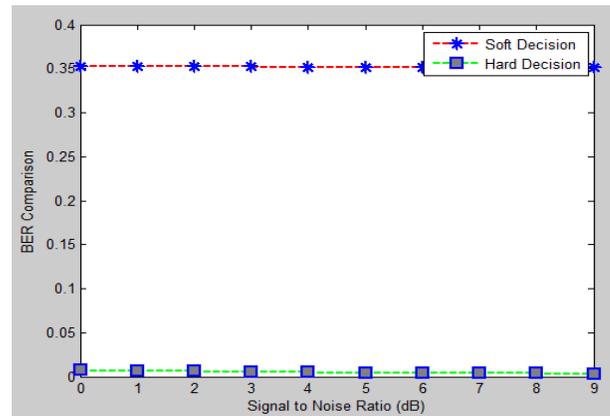

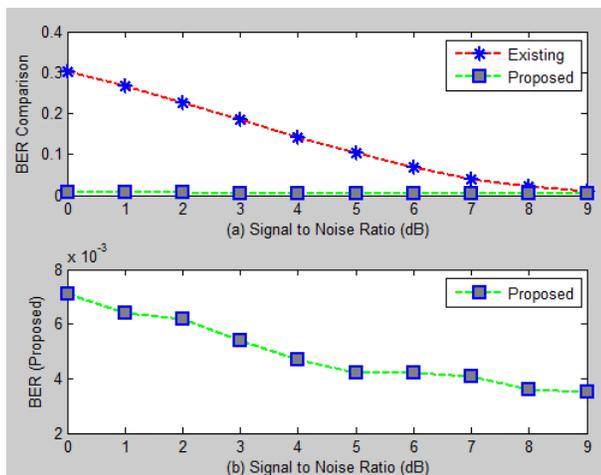**Figure 6 BER performance under varied decision based decoders**

Figure 5 Comparative BER performances of our proposed system and existing[23] approach

VII. CONCLUSION

Being a very potential candidate for mobile communication system, GSM has emerged with varied communication facilities and supporting services. The ultimate service qualities of GSM systems are undoubtedly influenced by the efficiency of modulation techniques and signal decoding. In addition, synchronization of signals can be a significant approach to enhance efficiency of the communication system. Considering these as motivation, in this paper, a Gaussian Multiple Shift Keying (GMSK) synchronization scheme has been developed for GSM systems. The novelty of this paper is the implementation of symbol time offset estimation, carrier frequency offset estimation and carrier phase offset estimation altogether for an efficient synchronization. Furthermore, the implementation of symbol by symbol (SBS) demodulator for signal decode has also resulted better performance.

The developed burst mode transmission paradigm based GSM system and its efficient synchronization has enabled it to be efficient for low SNR environment. This research affirms that a well defined and optimal combination of the proposed GMSK synchronization, SBS decoder and hard decision computation can ensure better result for GSM system. In future, the comparative performance analysis of the proposed scheme can be done with other modulation techniques and synchronization approaches for GSM systems.

REFERENCES

[1] "GSM Association, Market Data, www.gsmworld.com, August 2010.

[2] Cicalini A. et al., "A 65nm CMOS SoC with embedded hsdpa/edge transceiver, digital baseband and multimedia processor," in Solid-State Circuits Conference Digest of Technical Papers (ISSCC) IEEE International, 2011; pp. 368 –70.

[3] Benkeser C, Bubenhofer A, Q Huang, "A 4.5mW Digital Baseband Receiver for Level-A Evolved EDGE," in Solid-State Circuits Conference Digest of Technical Papers (ISSCC), IEEE International, 2010, pp. 276–77.

[4] Li Y, Sun S, Kwok Y S, "Energy Efficient Low-Complexity Symbol by Symbol GMSK Demodulator for BAN," accepted by IEEE VTC, spring, May, 2011; pp. 1-5 .

[5] Booth R. W. D, "An illustration of the MAP estimation method for deriving closed-loop phase tracking topologies: The MSK signal structure," IEEE Trans. Commun., COM-28, 1980; pp. 1137-42.

[6] Mehlan R., Y. Chen, and H. Meyr, "A fully digital feedforward MSK demodulator with joint frequency offset and symbol timing estimation for burst mode mobile radio," IEEE Trans. Vehicular Tech., 1993; vol. 42: pp. 434-43.

[7] M. Morelli and U. Mengali, "Joint frequency and timing recovery for MSK-type modulation," IEEE Trans. Commun., 1999; 47: pp. 938-47.

[8] Morelli M. and Vitetta G. M, "Joint phase and timing synchronization algorithms for MSK-type signals," IEEE Communication Theory Mini Conference, 1999; pp. 146-50.

[9] Huang Y. L, Fan K. D and Huang C. C, "A fully digital noncoherent and coherent GMSK receiver architecture with joint symbol timing error and frequency offset estimation," IEEE Trans. Vehicular Techno., 2000; 49: pp. 863-74.

[10] Li Y, Kwok Y, and Sun S, "Energy Efficient Low-Complexity Symbol by-Symbol GMSK Demodulator for BAN," IEEE VTC, 2011.

[11] Mehlan R, Chen Y, and Meyr H, "A fully digital feedforward MS demodulator with joint frequency offset and symbol timing estimation for burst mode mobile radio," IEEE Trans. Vehicular Tech., 1993; 42: pp. 434-43.

[12] Morelli M. and Mengali U, "Joint frequency and timing recovery for MSK-type modulation," IEEE Trans. Commun., 1999; 47: pp. 938-47.

[13] Morelli M. and Vitetta G. M "Joint phase and timing synchronization algorithms for MSK-type signals," IEEE Communication Theory Mini-Conference, 1999; pp. 146-150.

[14] Fitz M. P "Planer filtered techniques for burst mode carrier synchronization," in Proc. Globecom, 1991; 43: pp. 1169-78,

[15] Hagenauer J, "Soft-in/soft-out—the benefits of using soft values in all stages of digital receivers," in 3rd Int. Workshop Digital Signal Processing Techniques Applied Space Commun., 1992.

[16] Bahl L. R, Cocke J, Jelinek F, and Raviv J, "Optimal decoding of linear codes for minimizing symbol error rate," IEEE Trans. Inform. Theory, 1974; 20: pp. 284–7.

[17] Forney G. D, "The Viterbi algorithm," Proc. IEEE, 1973; 61: pp. 268–78.

[18] Lodge J. and Moher M, "Maximum likelihood sequence estimation of CPM signals transmitted over Rayleigh flat-fading channels," IEEE Trans. Commun., 1990; 38: pp. 787–794.

[19] Lodge J, Hoeher P, and Hagenauer J, "The decoding of multidimensional codes using separable MAP filtering," in Proc. 16th Biennial Symp. Comm., 1992; pp. 343–6.

[20] Lodge J, Young R, Hoeher P, and Hagenauer J, "Separable MAP 'filters' for the decoding of product and concatenated codes," in Proc. IEEE ICC'93 pp. 1740–45.

[21] Berrou C, Glavieux A, and Thitimajshima P, "Near Shannon limit error correcting coding and decoding: Turbo codes(1)," in Proc. IEEE ICC'93, pp. 1064–70.

[22] Papoulis A, Probability, Random Variables, and Stochastic Processes. New York: McGraw-Hill, 1984.

[23] Dharma D, and Sharma, A, "BER performance of GMSK using Matlab," International Journal of Advanced Research in Computer Engineering & Technology, 2013; 2: pp. 1989-92.