

Shell Elastic Critical Load Enhancement for Load Sensing Applications: Design Exploration and Study of Response Surface Characteristics

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Abstract— This paper studies the impact of contact feature modifications on the elastic critical load response, in the context of load sensing applications. Compact load sensing devices require the ability to withstand higher elastic loads without undergoing plastic deformation. New design variables are introduced to expand the design space of the hemispherical shell. A validated FE model is used to perform several numerical simulations to quantify and analyse the response characteristics. It is shown that choice of variables helps in significant increase in elastic load capacity while maintaining compactness. The response characteristics are described in detail.

Keywords—Shells, elastic critical load, design exploration, response surface

I. INTRODUCTION

Hemi-spherical shells subjected to compression by a rigid flat plate has been studied in the literature both experimentally and numerically focusing on elastic, elastic-plastic and buckling characteristics. A brief overview of key literature is presented below along with motivation for the present study.

Reissner[1] investigated the shell-plate contact problem and provide expressions for direct and bending stresses for shallow shells with $h/R < 4$. Naghdi[2] later extended Reissner's study to include the effect of shear strains. Kalnins[3] devised a multi-segment method for nonlinear analysis of elastic shells. Updike etal[4] investigated the load behavior of an compressed elastic shell, coming up with an analytical formulation for elastic load-deflection and also buckling phenomena wherein the shell deforms with an axisymmetric dimple at the center. Subsequent studies by Shwarz[5] and Kitching[6] focused on load-interference behavior as a function of shell thickness and radius. Interestingly, they were looking at contrasting shell applications, one concerning cornea and other collision of vehicles. Gupta etal[7] focused on buckling of hemispherical shells across a range of thickness ratios and showed good agreement between experimental and theoretical results.

Another interesting shell load application to ping pong ball was studied by Pauchard etal[8]. Shariati etal[9] performed experimental and numerical study of buckling behavior of steel shells with flat tops under various loadings. Recent studies on deformation behavior of hemispherical shells are captured in references [11] and [12].

In this paper, elastic critical load enhancement of shell is studied under different design modifications. Elastic critical load is the maximum load the shell can take without undergoing plastic deformation and hence is an important consideration for maximizing the potential of a load sensor. The modifications are designed to alter the contact features and also to investigate opportunity for compactness.

Understanding the physics of plastic yielding viz. dependence on geometrical/material properties and its initiation location helps in effecting design changes to maximize load capacity. A validated finite element model has been used to explore this design space to study the stress fields and quantify the impact of design parameters on elastic load capacity. The study of Longqiu Li etal[10] provides a pointer in using FE model to derive useful information on onset of plastic yielding but is limited to the treatment of spherical shells.

Next section shows details of the load device setup. To enable such an application of shell geometries, it is imperative to demonstrate high elastic load bearing capacity with a compact configuration and this forms the subject of present study.

II. CASE SETUP

The build-up of the shell load device is shown in Fig.1 and description is as follows. Multiple shell elements are placed between two rigid plates which form the outer covering of the device. Any load onto the rigid plates is transferred uniformly to all the shells which in turn undergo a given deformation. The piezoelectric transducer placed between the rigid plates serves to measure this deflection by a change in voltage which is calibrated to the load acting on the device.

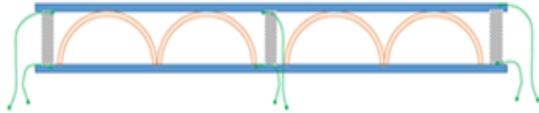


Fig 1: Shell Load Sensing Device

Piezoelectric transducers are placed at multiple locations to mitigate sensitivity to placement. As mentioned before, maximizing the load capacity of a single shell will reduce the footprint of the device by reducing count and hence weight of the device.

Fig 2 shows variants of shell arrangements in the device depending on the space and load requirement.

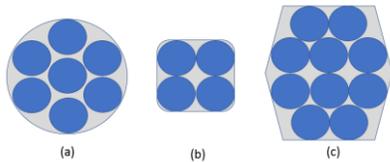


Fig 2: Possible shell arrangements (circular, square, hexagonal)

III. DESIGN STUDY

Conventional design parameters of shells studied in the literature include thickness (t), outer radius (R) and material properties such as Young's Modulus (E) and Yield strength (Y). In Ref [10], it has been shown that the load capacity of a conventional hemispherical shells can be related to a non-dimensional parameter λ , (Eq-1). It is reported that, for hemispherical shells, critical load attains a peak of 4X of solid sphere at $\lambda = 0.75$, where

$$\lambda = \log \left(\left(\frac{t}{R} \right) \left(\frac{E}{Y} \right)^{0.886} \right) \quad \text{Eq. (1)}$$

The current study aims to answer the following open questions:

- 1) Can an extended design space help in further increase of load capacity? If yes, by how much?
- 2) Can the shell be made more compact without losing the increased load capacity?

To facilitate, the following parameters have been introduced to expand the design space.

- **Radius of flat portion on top of the shell (r^*):** Intuitively speaking, the contact region of the shell will have a bearing on the stress distribution and hence elastic limit. However, the improvement in load capacity and interaction with other variables is to be determined.

- **Height of the shell (h):** Ability to adjust height can potentially help to make the shell more compact. However, it is to be seen if this has any adverse impact on load capacity.

Impact of these parameters on elastic critical load has not been studied in literature. Making these non-dimensional with respect to the outer shell radius, we now have the following:

- a) Flat-top Radius Ratio (r^*/R)
- b) Height ratio (h/R)

Fig.3 shows the consolidated design parameters (blue – conventional parameters & orange – new parameters)

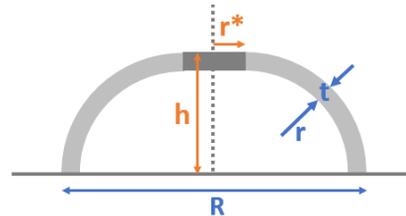


Fig 3: Consolidated shell design parameters

In addition to these geometry variables, material properties are also included in the design study. The DoE table with min/max limits of each variable is as shown below.

TABLE 1
RANGE OF CONSIDERED DESIGN VARIABLES

Variable	t/R	r^*/R	h/R
Min	0.02	0	0.5
Max	0.6	0.1	1

Before embarking on the studying the impact of new design parameters, a validation study is carried out with conventional parameters. Results quoted in Ref [10] have been successfully reproduced with the finite element model. In this process, iterations were also performed on mesh density to ensure the adequacy of the model for further design studies. Details of the FE model are shown in the next section.

A. Finite Element Model

The FE model is an axisymmetric shell between top and bottom rigid plates. The shell is fixed to the bottom rigid plate and hence it is constrained in vertical and axial directions as shown in Fig 4(a). On the top side, contact is simulated between the shell and the rigid plate. Only one half of the axi-symmetrical hemispherical shell is modeled exploiting symmetry. On the symmetry end, model is constrained in horizontal direction.

ANSYS axi-symmetric shell element PLANE183 is used to model shell & rigid plate. The top plate is made rigid by imposing young's modulus of 1000 times the shell material. Displacement is applied on the top rigid plate and is divided into 100 load steps for accurately capturing the non-linear contact and elastic-plastic transition. The typical mesh count of the model is ~35000 and it typically takes ~0.5-1 hour to run on a 2-core 3GHz CPU. The zoomed portion of the mesh near the contact region is also shown in the adjacent figure.

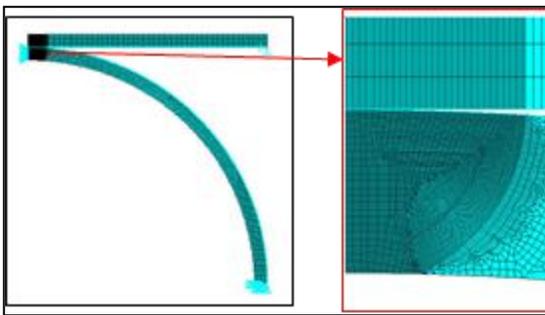


Fig. 4(a) FE model of hemispherical shell with zoomed view of the mesh near contact

APDL macros have been generated to run through design studies in an automated fashion. This helps in performing multiple tradeoff studies using desired number of parameters and facilitates focusing on results.

IV. RESULTS AND DISCUSSION

A total of 125 simulations have been performed with various combination of design parameters. Results will be discussed on two fronts, first being the effect of flat top and second the height of the shell.

A. Impact of flat top radius

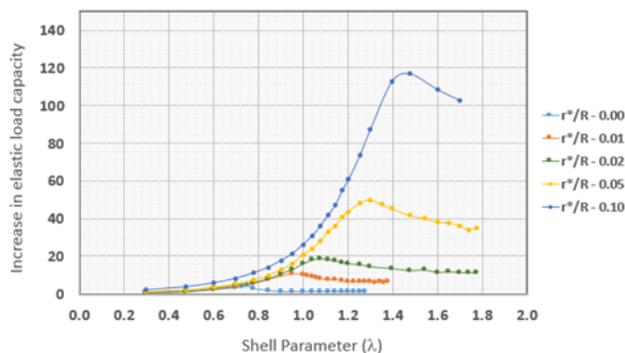


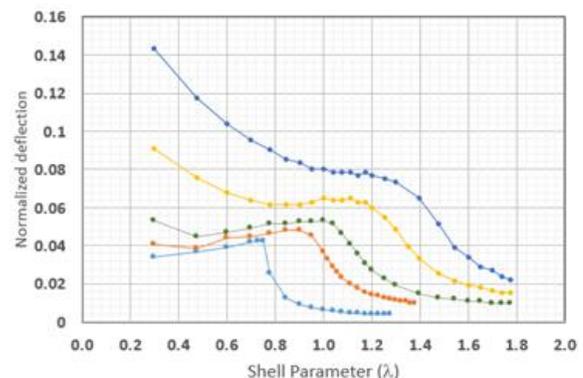
Fig 5. Impact of flat top radius on critical load capacity

The case of $r^*/R=0$ corresponds to the previous study on hemispherical shells. Current model reproduces the 4x increase in critical load at $\lambda=0.75$. Below are the few important characteristics of the shell response. Since the shell parameter has both material & geometric properties, we will see the impact of geometry for a given material.

- a) Introduction of flat top on hemispherical shell greatly helps in increased elastic limit, influencing the stress redistribution near the contact point.
- b) As the flat top radius increases, the peaking of critical load shifts to a higher thickness-ratio (for a given material), which points to higher weight of the device. This results in trade-off between load capacity and weight.
- c) The trend of increased load capacity is relatively unchanged for lower thickness-ratios ($\sim < 0.8$).
- d) At higher thickness ratios, the load capacity reaches a new mean value (as opposed to 1 for hemispherical configuration). This is discussed later in section 4.3.
- e) The peaking of critical load becomes a more gradual transition with the addition of flat top.

We now look at Fig-6 as to how the deflections are impacted by the addition of flat top. Deflection shows the following characteristics. Again here, the material properties are taken as constant to drive clear inferences on the impact of geometry change.

- a) Deflection is greatly increased only for shells with lower thickness ratios ($\sim < 0.8$)
- b) Smooth transition of peak critical load with the addition of flat top and its occurrence at larger thickness ratios can be clearly observed.
- c) Interestingly, for shells with high thickness ratios, all curves appear to converge to same deflection.



B. Impact of shell height

The above DoE is re-analysed for a reduced height to assess its impact. Fig 7 depicts a modified version of Fig 5 with the addition of load curve for reduced height (highlighted in red rectangle). It is seen that the global design change of height reduction has little or no impact on the elastic load capacity of the shell, thus providing an avenue for a compact design for applications where space is a major constraint.

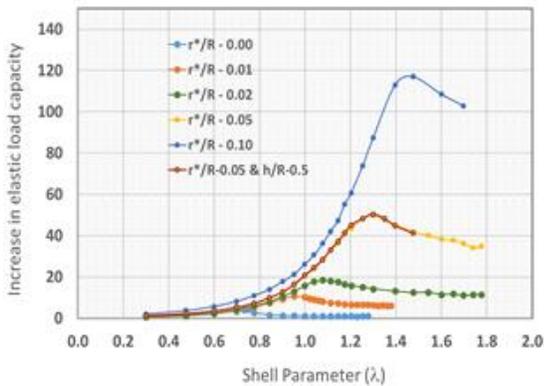


Fig 7. Comparison of load with baseline & 50% height

C. Generalised shell parameters

In summary, generalized shell parameters have been arrived at, leveraging the extended DoE in this current study.

The critical lambda (lambda at which the load attains peak capacity) increases with r^*/R according to the following relation (Eq-2) as shown in Fig 8. This extends the applicability of spherical shell parameter to the new design space and can be used to estimate the shell thickness for a particular choice of flat top radius and other design parameters.

$$\lambda - 0.75 = 0.4 \sqrt{\frac{r^*}{R}} \dots\dots\dots \text{Eq (2)}$$

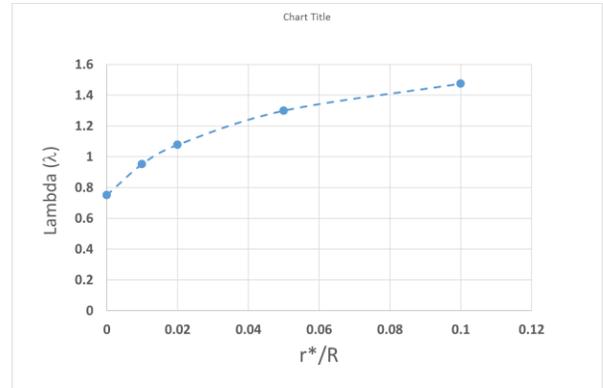


Fig 8. Influence of r^*/R on critical lambda

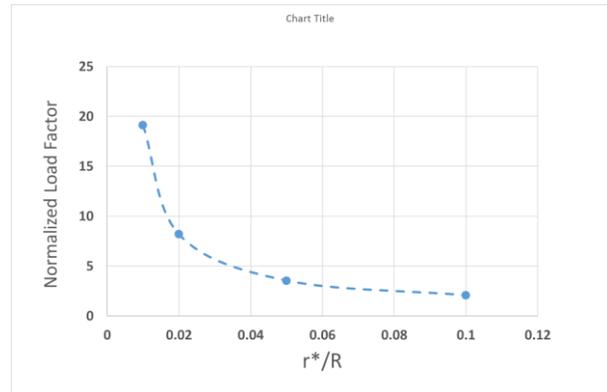


Fig 9. Influence of r^*/R on normalized load factor

It is instructive to look at load bearing capacity from a different perspective. It has been pointed earlier, that for flat top configurations, the load increase for higher thicknesses settles to a new average >1 . For flat top configurations, a new way of normalizing the load is proposed, designated as “Normalized Load Factor” It is defined as the peak load at a particular r^*/R normalized to the simple compressive load required for the shell to yield for that r^*/R . This parameter is plotted in Fig 9. For r^*/R of 0, the curve is at infinity due to infinitesimally small area of contact radius for spherical shell ($r^* \rightarrow 0$), while it tends towards the simple compressive load as the flat top radius is increased.

V. CONCLUSIONS

This paper highlights novel application of shell as load measuring device and explores design study to maximize the load bearing capacity of the shell. Impact of load capacity is quantified and contributions of newly added design variables has been shown to help in compact design of the shell while significantly enhancing the elastic limit. Several characteristics of the elastic limit increase are described in detail in terms of variation with thickness and the flat top radius. Ability to reduce the height of the shell without losing elastic load capacity will result in compact configuration. Continuing this research, experiments are planned with the shells to establish realization factor and study the fabrication and instrumentation aspect of the load sensors.

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