

Buckling Analysis of Laminated Composite Beam

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Abstract- Buckling of plates is a well-established branch of research in composite structures stability. The present study, deals with effects of stacking sequences, thickness, length, fiber orientation, length to thickness ratio, aspect ratio and various boundary conditions on the critical buckling loads of the laminated composite beams of different fibers. ANSYS composite Prepost is used as the tool for modelling and analysis of the composites. Experiments are conducted for validating the analysis results. This model will reveal many details and properties of E-Glass fibre reinforced composite materials. The proposed study is to be carried out by finite element analysis procedure and validating it with buckling test key-results of the test specimen.

Keywords —Composite Laminate, FEM, Hand layup fabrication, Buckling Analysis

I. INTRODUCTION

In many engineering structures like columns, beams, or plates etc., their failure developing not only from higher stresses but also from buckling of the material. Only rectangular thin plates are considered in the study. When a flat plate is subjected to low in-plane compressive loads or stress, it remains flat and is in the equilibrium condition. As the magnitudes of the in-plane compressive loads or other loads increases, then equilibrium configuration of the plate is eventually deformed to a non-flat configuration and the plate becomes unstable and may be deformed. The magnitude of the compressive loads at which the plate becomes unstable or deformed is called the “critical buckling load.” A composite material having two or more materials and offering the weight saving in structures in view of its high strength, weight, and high stiffness ratios. Further, in a fibrous composite, the mechanical properties may be varied as required by suitably orienting the fibers. In this kind of material the fibers carry major load carrying members, and the matrix, which has low modulus and high elongation, provides the high flexibility and also keeps the fibers in position and protect them from the environment. Development of newly introduced applications and new composites materials is accelerating due to the requirement of materials with a combination of properties that cannot be met by ordinary monolithic materials. Actually, composite materials are capable fulfilling this requirement by all means because of their heterogeneous nature.

Properties of composite material introduced as a function of its components materials, their distribution and the interaction among them and results an unusual combination of composite material properties can be obtained.

Laminated composites materials are having wider use in mechanical and aerospace applications because of their high specific stiffness, high specific strength, and less weights. Fiber-reinforced composites materials are using widely in the form of a relatively thin plate, beams and consequently, the weight carrying capability of composite materials plate or beam against the buckling load has been considered by scientists and researchers under various loading and boundary conditions. They have excellent stiffness and weight characteristics, so composites material have been receiving more priority from engineers, scientists, and designers. In many applications the composite laminate plates are commonly subjected to loads likes compression loads, which may cause buckling failure if overloaded. So that their buckling load's behaviors are very important factors in the safe and reliable design of these structures.

II. LITERATURE REVIEW

Radoslaw J. Mania and Christopher Bronn York [1] investigated strength improvements for fiber metal laminates using thin ply tailoring method. The comparison was made between FRP layers versus thin ply technology designs, applied to fiber metal laminate plate structure. Also, comparisons were made between classical glass fiber reinforced plastic and thin ply carbon fiber reinforced plastic. Controlling the bending stiffness through appropriate laminate tailoring strategies, and material and ply thickness selection has been shown to give improvements in the compressive buckling load capacity for fiber metal laminate short columns of an open cross section. L Santosh Sreekanth [2] investigated composite beam with transverse crack and found that buckling load decreases as the depth of crack at any particular location increases. Also buckling loads decrease with increase in angle of fiber. Comparison of GFRP and EFRP was carried out and observed that rate of change of buckling load of GFRP is more than EFRP.

Ashish Desai and N.K. Chhapkhane [3] investigated structural analysis of composite beam having I-cross section under transverse loading. It was observed that stress in composite laminate was less than the conventional material followed by weight reduction of composite. Goikmen Atihan [4] researched on effect of various parameters of laminate composite. It was observed that buckling load varies with changing boundary condition, stacking sequence and fiber orientation angle. Prabhuling Sarasambi [5] has worked on to identify better configuration of given composite to achieve higher buckling strength for laminated composite structures subjected to uniaxial compressive load. It was found that increasing the number of lamina the critical buckling load of laminate also increases. Y.X Zhang[7] reviewed more than 120 papers and found that research was mainly carried out on various lamination theories, with the focus on dynamic analysis, geometric nonlinearity and large deformation analysis and failure and damage analysis of composite laminated plates. But it has been found that aspects of the research were limited and may attract more interests in future research. Buket Okutan Baba [8] carried out a numerical and experimental study out to determine the effects of anti-symmetric laminate configuration, cutout and length/thickness ratio on the buckling behavior of E/glass-epoxy composite plates. It was observed that experimental buckling loads are higher than predicted buckling loads due to initial imperfections of the specimens. Anti-symmetric laminates are having higher buckling strength compared to symmetric laminates.

III. PROBLEM STATEMENT

From the literature review, it was found that most of the studies were focused on unidirectional fiber. Industry driven woven fibers are being increasingly used in many industries. Hence we have to give more importance to its structural behavior. It also indicates that the interaction among stacking sequence, fiber thickness and length/thickness ratio on the buckling behavior of woven fiber laminated composites are needed to investigate in more detail. The aim of performing this research is to extend the knowledge of the structural behavior of woven fabric composites subject to compressive load which is lacking. The main objective of this study is to carry out the experiment analysis for the woven glass-epoxy composite laminated plates with subjected to the static compressive load.

IV. OBJECTIVES

Thus far there have been numerous studies on the E-glass fiber composite laminated structures which find widespread applications in a lot of engineering areas such as aerospace, biomedical, civil, and marine and mechanical engineering because they are easy of handling, good mechanical properties and low making cost. They also have excellent damage tolerance and impact resistance. It is clear that most of the studies and investigations are based mainly on the numerical approaches. Very less attention has been paid on the buckling of composite plates.

Most of the research studies were focused on unidirectional type fiber. Industrial driven woven type fibers are being increasingly used in many applications. So we have to give more importance to its structural behavior also. The main aim of undergoing this research is to understand structural behavior of woven type fabric composites subject to compressive load.

- To find out the experiment of buckling load analysis of laminated E Glass fiber composite beam using ANSYS.
- To understand the role of woven fiber in buckling analysis
- Analyzing effect of length, thickness, stacking sequence, fiber orientation and lamina thickness on buckling strength of laminate composite.
- Understand the FEA procedure for laminate composite in order to compare it with theoretical procedure.

V. MATHEMATICAL FORMULATION

Basic Assumption of classical laminate plate theory

Following assumption are made for classical laminate theory

- a) Thickness is much smaller in compression to other physical dimensions.
- b) The principal material direction of each layer need not coincide with plate axes.
- c) Behavior of each layer is linear and elastic.
- d) Each ply and the beam have constant thickness.
- e) Transverse normal stress is neglected.
- f) Transverse normal strains are neglected.
- g) Transverse shear stresses are assumed to be zero.

ANSYS is used to carry out the finite element analysis in the work. ANSYS is used to analyze the buckling load Of Woven glass fiber composite beam of different sizes.

The dimensions of the specimen are 150*20*3mm in length, width, and thickness. Eigen value buckling analysis in ANSYS has four steps:

- i. *Build the required model*: It includes defining element type, size, real constants, material properties and modeling.
- ii. *Solution (Static Analysis)*: It includes applying boundary conditions, applying loads and solving the static analysis. The applied boundary condition and the load is shown below
- iii. *Eigen buckling analysis*: Eigen value buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure.
- iv. *Postprocessor*: This step includes listing different buckling loads and viewing in different buckled shapes. We can plot the deformed and undeformed shape of the plate. The model created by using 5 plies of carbon fiber sheets. Each plies carries a thickness of 3mm and ply orientation [0, 90]

A. Classical Lamination Theory from Classical Plate Theory [6]

The classical lamination theory is almost identical to the classical plate theory, the only difference is in the material properties (stress-strain relations). The classical plate theory usually assumes that the material is isotropic, while a fiber reinforced composite laminate with multiple layers (plies) may have more complicated stress-strain relations.

The outcome of each of these segments is summarized as follows:

Kinematics:

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{pmatrix} + z \begin{pmatrix} k_x \\ k_y \\ k_{xy} \end{pmatrix} \quad (1)$$

Where, ε_x^0 , ε_y^0 , γ_{xy}^0 are called the mid-surface strains. They represent the stretching and shear of the plate, and are defined as

$$\begin{aligned} \varepsilon_x^0(x,y) &= \frac{\partial u_0}{\partial x} \\ \varepsilon_y^0(x,y) &= \frac{\partial v_0}{\partial y} \\ \gamma_{xy}^0(x,y) &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \quad \dots (2) \end{aligned}$$

Resultants:

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = \int_{-t/2}^{t/2} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix} dz \quad \dots (3)$$

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \int_{-t/2}^{t/2} \begin{pmatrix} \sigma_{yz} \\ \sigma_{xz} \end{pmatrix} dz$$

$$\begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = \int_{-t/2}^{t/2} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix} z dz \quad \dots (4)$$

Where N_x , N_y , and N_{xy} are the tensile and shear forces per unit length along the boundary of the plate element with units [N/m], V_x , and V_y are the shear forces per unit length of the plate with units [N/m], and M_x , M_y , and M_{xy} are the moments per unit length with units [N].

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = \sum_{k=1}^N \int_{z_{k-1}}^{z_k} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}^k dz \quad \dots (5)$$

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \sum_{k=1}^N \int_{z_{k-1}}^{z_k} \begin{pmatrix} \sigma_{yz} \\ \sigma_{xz} \end{pmatrix}^k dz \quad \dots (6)$$

$$\begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = \sum_{k=1}^N \int_{z_{k-1}}^{z_k} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}^k z dz \quad \dots (7)$$

Again, the subscript k indicates the k^{th} layer from the top of the laminate and N is the total number of layers. Note that perfect bonding is assumed so we can move the integration inside the summation.

Equilibrium

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = -P_x \quad \dots (8)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = -P_y \quad \dots (9)$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -P_z \quad \dots (10)$$

B. Forming Stiffness Matrix: A, B, and D

The plate is assumed to be constructed by a homogeneous but not necessarily isotropic material and subjected to both transverse and in-plan loadings. Also, the Cartesian coordinate system is used.

The goal is to develop the relations between the external loadings and the displacements. However, the relations between the resultants (forces N and moments M) and the strains (strains ϵ and curvatures κ) are of most interest in practice.

Replace the stresses in the force and moment resultants with strains via the constitutive equations, we have

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{pmatrix}^k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}^k \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_6 \end{pmatrix}^k \quad \dots (11)$$

$$\begin{pmatrix} \sigma_4 \\ \sigma_5 \end{pmatrix}^k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix}^k \begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix}^k \quad \dots (12)$$

Where the superscript k indicates the lamina number.

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^k \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{pmatrix}^k \quad \dots (13)$$

$$\begin{pmatrix} \sigma_{yz} \\ \sigma_{xz} \end{pmatrix}^k = \begin{bmatrix} \bar{Q}_{44}^* & \bar{Q}_{45}^* \\ \bar{Q}_{45}^* & \bar{Q}_{55}^* \end{bmatrix}^k \begin{pmatrix} \gamma_{yz} \\ \gamma_{xz} \end{pmatrix}^k \quad \dots (14)$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad \dots (15)$$

$$\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = \begin{bmatrix} H_{44} & H_{45} \\ H_{45} & H_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad \dots (16)$$

Where A is called the extensional stiffness, B is called the coupling stiffness, and D is called the bending stiffness of the laminate. The components of these three stiffness matrices are defined as follows:

$$A_{ij} = \sum_{k=1}^N (\bar{Q}_{ij})_k (h_k - h_{k-1}) = \sum_{k=1}^N (\bar{Q}_{ij})_k t_k \quad \dots (17)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^N (\bar{Q}_{ij})_k (h_k^2 - h_{k-1}^2) = \sum_{k=1}^N (\bar{Q}_{ij})_k t_k \bar{h}_k \dots (18)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (h_k^3 - h_{k-1}^3) = \sum_{k=1}^N (\bar{Q}_{ij})_k \left(t_k \bar{h}_k^2 + \frac{t_k^3}{12} \right) \dots (19)$$

$$H_{ij} = \frac{5}{4} \sum_{k=1}^N (\bar{Q}_{ij}^*)_k \left[t_k - \frac{4}{t^2} \left(t_k \bar{h}_k^2 + \frac{t_k^3}{12} \right) \right] \quad \dots (20)$$

Where t_k is the thickness of the k^{th} layer and \bar{z}_k is the distance from the mid-plan to the centroid of the k^{th} layer. Forming these three stiffness matrices A , B , and D , is probably the most crucial step in the analysis of composite laminates.

VI. FINITE ELEMENT ANALYSIS

A. Materials and Methods

Materials used for making laminate composites are E-Glass fiber woven fabric as reinforcement and epoxy resin is used as matrix. The reinforcement is desired in only one direction and maintaining the ease of handling and drape properties of Woven fabric, so it is called uniaxial woven fabric. The design of laminated composite material using woven fabric lamina is similar to that of designing unidirectional lamina.

Woven fiberglass fabrics offer the widest range and the best control over thickness, weight and strength of all forms of fiberglass textiles. This offers the materials engineer a wide choice of controlled fabric properties to satisfy design needs and objectives.

The main properties of Woven Glass fabrics are,

- High Tensile Strength
- Dimensional Stability
- Durability
- Economical
- Good Chemical Resistance

These properties make them very famous in aerospace, civil engineering, military, and motorsports, along with other sports materials. And, they are relatively low in cost as compared to other fabrics such as roving. Fibers are used in composites because they are lightweight, stiff and strong. Fibers are stronger than the bulk material from which they are made. The high tensile strength of glass fibers is attributed to the low number and size of defects on the surface of the fiber. E glass is the preferred structural reinforcement because of the combination of mechanical performance, corrosion resistance, and low cost.

B. Model Analysis of Specimen

Specimen Dimensions: 150mm x 20mm x 3mm in Length width and thickness.



Fig.1. Composite Beam Specimen (150x20x3)

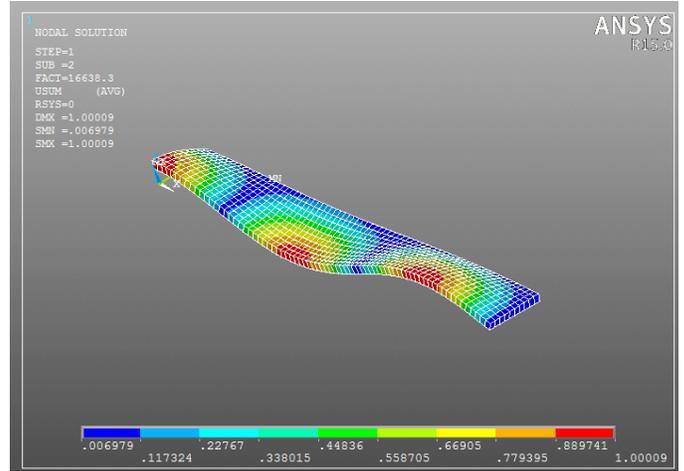


Fig.3. Buckling mode 2

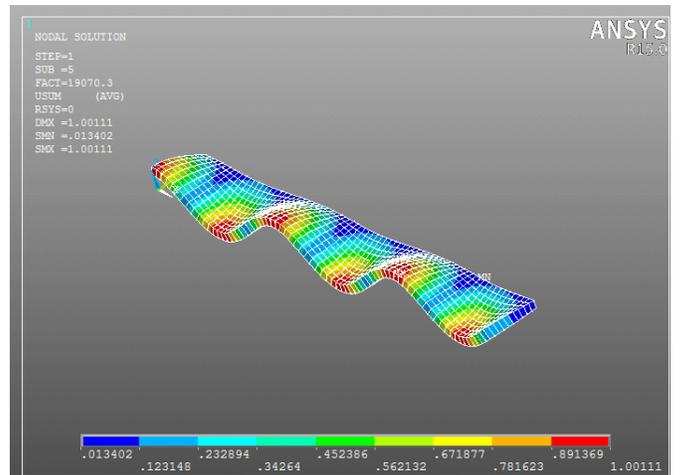


Fig.4. Buckling mode 3

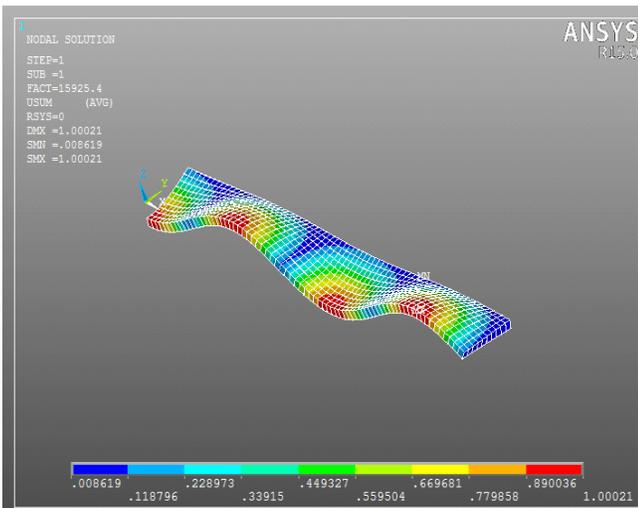


Fig.2. Buckling mode 1

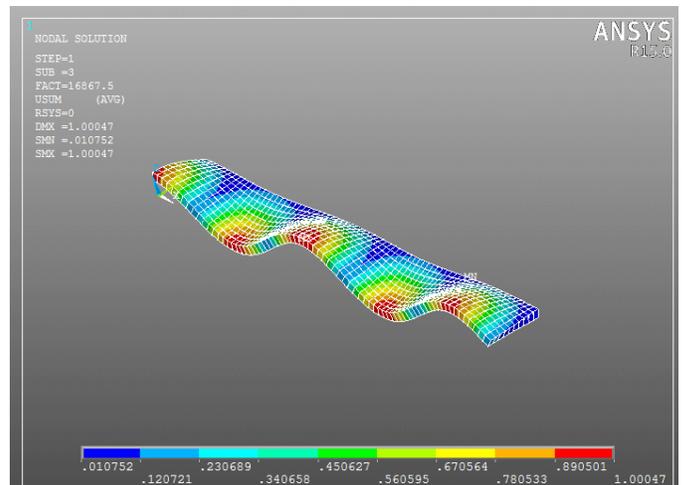


Fig.5. Buckling mode 4

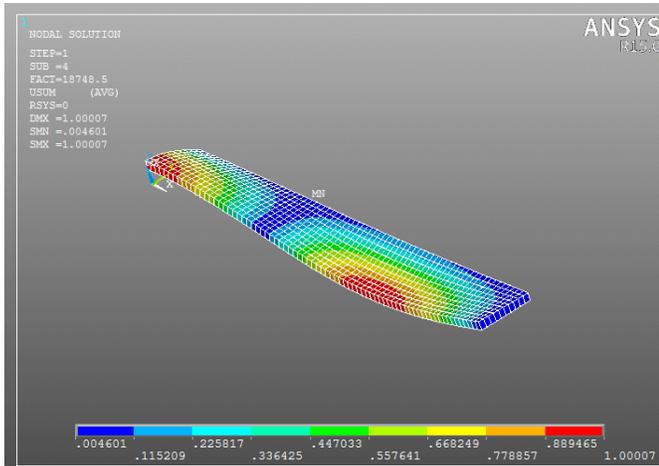


Fig.6. Buckling mode 5

**TABLE 1.
Buckling Loads And Deflections**

Eigen Value Buckling Deformation No	Maximum Deformation (mm)	Buckling Load (N)	Buckling Load (KN)
1	1.00021	15925.4	15.925
2	1.00009	16638.3	16.638
3	1.00047	16867.5	16.867
4	1.00007	18748.5	18.748
5	1.00111	19070.3	19.070

VII. EXPERIMENTAL STUDY

In the view of the difficulty of theoretical and numerical analysis for laminated structure behaviors, experimental methods are important to solve the buckling problem of laminated carbon fiber composite plates. To find out that fabricated composite plate's compression buckling test conducted. The test specimen was clamped on two sides and specimens were loaded in axial compression by using a compression testing machine of 600-tonne load capacity. In this buckling compression tests buckling load of woven glass fiber composite beam is determined.

A. Test Procedure

The fabricated woven fabric laminate composite was loaded in axial compression using a Universal Testing Machine of 600 KN capacity. Clamped boundary conditions were simulated along the top and bottom edges, restraining 40mm length.

For axial loading, the test specimens were placed between the two extremely stiff machine heads, of which the lower one was fixed during the test, whereas the upper head was moved downwards by servo hydraulic cylinder. A dial gauge was placed at the center of the specimen to find out the lateral deflection. All plates of different dimensions were loaded slowly until buckling. As the compression load increased the centrally placed dial gauge needle started moving, and due to buckling there was a sudden movement of the needle. The compression load at this point will be the buckling load of the specimens.



Fig.7. Universal Testing Machine

VIII. RESULTS AND DISCUSSION

**TABLE 2.
Experimental Buckling Load Versus Ansys Buckling Load**

Dimension of Beam	Deflection (mm) measure by Dial Indicator	Experimental Buckling Load (KN)	ANSYS Bucklin Load (KN)
150*20*3	1	18.475	15.925
	2	19.121	16.638
	3	20.184	16.867
	4	22.488	18.748
	5	23.252	19.070

In the above result table the deflection is measured from the dial indicator placed at the center of the beam and load is taken from the Universal Testing Machine panel. The experimental results converge with ANSYS results with percentage error ranging from 10 to 20%, which are in acceptable range. The errors in the results are due to greater stiffness of elements in ANSYS.

TABLE 3.
Various Buckling Loads For Different Dimensions

Plate No.	Length (mm)	Thickness (mm)	Width (mm)	ANSYS Buckling Load (KN)
1	150	3	20	15.925
2	150	4	20	16.586
3	175	3	20	12.451

From the above table it is understandable that both experimental buckling load and Ansys analyzed buckling load are almost equal. Plates with different types of dimensions are used extensively due to different design requirements. Thus, the buckling load response of plates with different dimensions must be fully understood in the structural design. In this section, the effects of length and width with same thickness are taken in to account. The analysis indicates that the variation of the buckling loads is very sensitive to the length. It can be seen that buckling load generally decreases with increase in length.

IX. CONCLUSION

Plates with different types of dimensions are used extensively due to different design requirements. Thus, the buckling load response of plates with different dimensions must be fully understood in the structural design. In this section, the effects of length and width with same thickness are taken into account. Analysis results are obtained for different fiber orientation, boundary condition, thickness, length for the composite beam. It gives conclusion that critical buckling loads increases with increasing number of lamina.

It is seen that simply supported boundary condition have higher critical buckling loads than other boundary condition. The length to width ratio affects the load carrying capacity of composite beams. The results found in experimentally and analytically on simple supported beam is having difference with acceptable range.

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