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Integral Solutions of Homogeneous Biquadratic Diophantine Equation $x^4 - y^4 = 40\,(z+w)\,p^3$ With Five Unknowns

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***Abstract*-- The homogeneous biquadratic Diophantine equation given by $x^4 - y^4 = 40(z+w)\,p^3$ is analyzed for its patterns of non-zero distinct integral solutions. A few interesting relations between the solutions and special polygonal numbers are exhibited.**

***Keywords:* Homogeneous Equation, Integral Solutions, Polygonal Numbers, Pyramidal Numbers and Special Numbers**

I. Introduction

Biquadratic Diophantine equations homogenous and non–homogenous are rich in variety [1-3]. For an extensive review of sizable literature and various problems, one may refer [4-18]. In this communication, we consider yet another interesting biquadratic equation $x^4 - y^4 = 40\,(z+w)\,p^3$ and obtain infinitely many non-trivial integral solutions. A few interesting relations between the solutions and special polygonal numbers are presented.

Notations Used

- $t_{m,n}$- Polygonal number of rank 'n' with size 'm'
- CP_n^6 - Centered hexagonal Pyramidal number of rank n
- Gno_n – Gnomic number of rank 'n'
- FN_n^4– Figurative number of rank 'n' with size 'm'
- Pr_n - Pronic number of rank 'n'
- P_n^m- Pyramidal number of rank 'n' with size 'm'
- $carl_n$ -- Carol number
- ky_n - Keynea number
- Tha_n - Thabit number
- S_n – Star number of rank n
- SO_n - Stella octagonal number of rank n

II. Method Of Analysis

The Diophantine equation representing the biquadratic equation with five unknowns to be solved for its non zero distinct integral solutions is

$$x^4 - y^4 = 40(z+w)p^3 \quad (1)$$

Consider the transformation

$$\left.\begin{aligned} x &= u+v \\ y &= u-v \\ z &= uv+1 \\ w &= uv-1 \end{aligned}\right\} \quad (2)$$

On substituting (2) in (1) , we get

$$u^2 + v^2 = 10\,p^3 \quad (3)$$

In what follows, We illustrate methods of obtaining non Zero distinct integer solutions to (1)

Pattern I

Assume

$$p = A^2 + B^2 = (A+iB)(A-iB) \quad (4)$$

Equation (3) as

$$u^2 + v^2 = 10p^3$$

Where $10 = (1+3i)(1-3i)$ (5)

Using (4) *and* (5) *in* (3) and writing (3) in factorization form as

$$(u+iv)(u-iv) = (1+3i)(1-3i)[(A+iB)(A-iB)]^3$$

which is equivalent to the system of equation

$$u + iv = (A^3 + 3B^3 - 9A^2B - 3AB^2) + i(3A^3 - B^3 + 3A^2B - 9AB^2)$$

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Equating real and imaginary parts ,we get

$$u = (A^3 + 3B^3 - 9A^2B - 3AB^2)$$

$$v = (3A^3 - B^3 + 3A^2B - 9AB^2)$$

On substituting the values of u and v in (2)the nonzero distinct integral values of x, y,z,w and p satisfying (1) are given by

$$\left.\begin{array}{c} x = x(A,B) = 4A^3 + 2B^3 - 6A^2B - 12AB^2 \\ y = y(A,B) = -2A^3 + 4B^3 - 12A^2B + 6AB^2 \\ z = z(A,B) = 3A^6 - 24A^5B - 45A^4B^2 + 80A^3B^3 + 45A^2B^4 - 24AB^5 - 3A^6 + 1 \\ w = w(A,B) = 3A^6 - 24A^5B - 45A^4B^2 + 80A^3B^3 + 45A^2B^4 - 24AB^5 - 3A^6 - 1 \\ p(A,B) = A^2 + B^2 \end{array}\right\}$$

Properties

1. $2x(1,B) - y(1,B) + 30t_{4,B} - 10 \equiv 0$
2. $x(1,B) + 2y(1,B) - 5SO_B + 25B \equiv 0$
3. $z(1,1) + w(1,1) - 2p(1,1)$ is a nasty number
4. $z(2,-2) + 2p(1,1)$ is a perfect number
5. $x(2,2)$ is a nasty number

Pattern II

Equation (3) as

$$u^2 + v^2 = 10p^3$$

Where $10 = (3+i)(3-i)$ (6)

Using $(4) and (5) in (3)$ and writing (3) in factorization form as

$$(u+iv)(u-iv) = (3+i)(3-i)[(A+iB)(A-iB)]^3$$

which is equivalent to the system of equation

$$u + iv = (3A^3 + B^3 - 3A^2B - 9AB^2) + i(A^3 - 3B^3 + 9A^2B - 3AB^2)$$

Equating real and imaginary parts ,we get

$$u = (3A^3 + B^3 - 3A^2B - 9AB^2)$$

$$v = (A^3 - 3B^3 + 9A^2B - 3AB^2)$$

On substituting the values of u and v in (2) the nonzero distinct integral values of x, y, z, w and p satisfying (1) are given by

$$\left.\begin{array}{c} x = x(A,B) = 4A^3 - 2B^3 + 6A^2B - 12AB^2 \\ y = y(A,B) = 2A^3 + 4B^3 - 12A^2B - 6AB^2 \\ z = z(A,B) = 3A^6 + 24A^5B - 45A^4B^2 - 80A^3B^3 + 45A^2 + 24AB^5 - 3B^6 + 1 \\ w = w(A,B) = 3A^6 + 24A^5B - 45A^4B^2 - 80A^3B^3 + 45A^2 + 24AB^5 - 3B^6 - 1 \\ p(A,B) = A^2 + B^2 \end{array}\right\}$$

Properties

1. $x(A,1) + 2y(A,1) - 10Hex_n - 10\, t_{4,A} \equiv 0$
2. $2x(A,1) + y(A,1) - 10CP_A^6 + 30A \equiv 0$

Pattern III

Equation (3) as

$$u^2 + v^2 = 10p^3 * 1 \qquad (7)$$

Write 1 as

$$1 = \frac{(A^2-B^2+2iAB)(A^2-B^2-2iAB)}{(A^2+B^2)^2} \qquad (8)$$

Using $(4) and\ (8)\ in\ (3)$ and writing (3) in factorization form as

$$(u+iv)(u-iv) = (1+3i)(1-3i)[(A+iB)(A-iB)]^3 \frac{(A^2-B^2+2iAB)(A^2-B^2-2iAB)}{(A^2+B^2)^2}$$

which is equivalent to the system of equation

$$u + iv = (1+3i)(A+iB)^3 \frac{(A^2-B^2+2iAB)}{(A^2+B^2)}$$

Equating real and imaginary parts ,we get

$$u = \frac{1}{(A^2+B^2)}(A^5 - 15A^4B - 10A^3B^2 + 30A^2B^3 + 5AB^4 - 3B^5)$$

$$v = \frac{1}{(A^2+B^2)}(3A^5 + 5A^4B - 30A^3B^2 - 10A^2B^3 + 15AB^4 + B^5)$$

On substituting the values of u and v in (2)the nonzero distinct integral values of x, y,z,w and p satisfying (1) are given by

$$\left.\begin{array}{c} x = x(A,B) = \frac{1}{(A^2+B^2)}(4A^5 - 10A^4B - 40A^3B^2 + 20A^2B^3 + 20AB^4 - 2B^5) \\ y = y(A,B) = \frac{1}{(A^2+B^2)}(-2A^5 - 20A^4B + 20A^3B^2 + 40A^2B^3 - 10AB^4 - 4B^5) \\ z = z(A,B) = \frac{1}{(A^2+B^2)^2}(3A^{10} - 40A^9B - 135A^8B^2 + 480A^7B^3 + 630A^6B^4 - 1008A^5B^5 - 630A^4B^6 + 480A^3B^7 + 135A^2B^8 - 40AB^9 - 3B^{10}) + 1 \\ w = w(A,B) = \frac{1}{(A^2+B^2)^2}(3A^{10} - 40A^9B - 135A^8B^2 + 480A^7B^3 + 630A^6B^4 - 1008A^5B^5 - 630A^4B^6 + 480A^3B^7 + 135A^2B^8 - 40AB^9 - 3B^{10}) - 1 \\ p = p(A,B) = A^2 + B^2 \end{array}\right\}$$

Properties

1. $z(A,0) - 3A^6 - 1 \equiv 0$
2. $p(0,B) - 2t_{4,B}y \equiv 0$
3. $p(2^n, 2^n) = Carl_n + Ky_n + 2$
4. $2p(A,A)$ is a nasty number

III. Conclusion

In this paper, we have presented four different patterns of non- zero distinct integer solutions of biquadratic Diophantine equation $x^4 - y^4 = 40\,(z+w)p^3$ and relations between solutions and special numbers are also obtained. To conclude, one may search for other patterns of solutions and their corresponding properties.

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