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Edge Colouring in Fuzzy Graph Structures

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***Abstract*— In this paper, we consider Wheel and Bistar graph and assign membership function for the edges of these graphs. Fuzzy graph structures for these graphs are constructed. R-independent edge colouring number χ_{R_i} of a fuzzy graph structure and the strong R-edge colouring number χ_{R_s} of a fuzzy graph structure are computed.**

Keywords*— Graph structures, Coloring, Edge coloring, Chromatic index

I. INTRODUCTION

The notion of fuzzy sets was introduced by L.A.Zadeh [17] in 1965 which paved way to development of Fuzzy graph theory. The first definition of Fuzzy Graph was introduced by Kaufmann [4] in1973 and then it was developed by Azriel Rosenfeld [3] in 1975. The concept of fuzzy graphs was discussed by Mordeson and Nair [7], Bhattacharya [2], Sunitha and Vijayakumar (2002) and so on. Graph structure concept was introduced by E.Sampath kumar [16] in 2006. Later Fuzzy graph structures were introduced by Ramakrishnan and T.Dinesh [13, 14, 15] in 2011. Colouring of fuzzy graphs was developed by S.Munoz, T.Ortuno, J.Ramirez and J.Yanez[8]. Later by S.Lavanya and R.Sattanathan [5,6].

II. METHODOLOGY

Let $G\ (V,E)$ be a graph with vertex set V and edge set E. This G becomes a fuzzy graph [9] when membership function is assigned for vertices and edges as $\rho: V \to [0,1]$ and $\mu: V \times V \to [0,1]$. Here, the membership function for edges is assigned following a certain pattern. Then the edges with same membership function are grouped as $R_1, R_2 \dots R_k$ [10, 11, 12]. Then we obtain a fuzzy graph structure $\tilde{G}(V, R_1, R_2 \dots R_k)$. For this fuzzy graph structure $\tilde{G}$, R-independent edge colouring number χ_{R_i} of a fuzzy graph structure and the strong R-edge colouring number $\chi_{R_i}$of a fuzzy graph structure are obtained.A set of edges in a fuzzy graph structure is R-independent if for any edge in R_i is not a subset of end vertices of some R_j edge $(i \neq j)$.

A set of edges in a fuzzy graph structure is strongly $R-$independent if for any edge in R_i is not a subset of the union of end vertices of different R_j edges$(i \neq j)$.

The R-independent edge colouring number $\chi_{R_i}$of a fuzzy graph structure $\tilde{G}$ is minimum order of the partition of edge set into R-independent sets.

The strong R-edge colouring number χ_{R_s} of a fuzzy graph structure $\tilde{G}$ is minimum order of the partition of edge set into strong R-independent sets.

For any fuzzy graph structure $\chi_{R_i}(\tilde{G}) \leq \chi_{R_s}(\tilde{G})$

Theorem1.1: For the membership function defined for the edges fuzzy graph structure of wheel graph $\widetilde{W}$, $\chi_{R_i}(\widetilde{W}) = 2$, $\chi_{R_s}(\widetilde{W}) = n$.

Proof:

Consider,$W_n\ (C_n + K_1\ is\ the\ wheel\ with\ n + 1\ vertices\ and\ 2n\ edges$)(Fig.1)

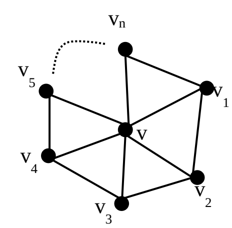

Figure 1

Let $\mu(v_2, v_3) = x_1$, $\mu(v_3, v_4) = x_2$, $\mu(v_4, v_5) = x_3, \dots$, $\mu(v_{n-1}, v_n) = x_{n-2}$,$\mu(v_n, v_1) = x_{n-1}$, $\mu(v_1, v_2) = x_n$
$\mu(v, v_1) = x_1, \mu(v, v_2) = x_2, \mu(v, v_3) = x_3, \dots$,
$\mu(v, v_{n-1}) = x_{n-1}$, $\mu(v, v_n) = x_n$

The value of $x_1, x_2, x_3 \dots x_{n-1}, x_n$ lies in [0,1] and $\{x_1, x_2, x_3, \dots, x_{n-1}, x_n\}$ is strictly non decreasing.

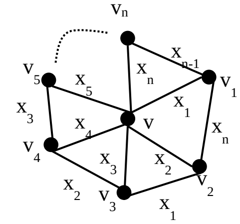

Figure 2

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Group the edges with same membership functions

$R_1 = \{(v_2,v_3),(v,v_1)\}$,
$R_2 = \{(v_3,v_4),(v,v_2)\}$,
$R_3 = \{(v_4,v_5),(v,v_3)\}$,
$\vdots$
$R_{n-1} = \{(v_n,v_1),(v,v_{n-1}),$
$R_n = \{(v_1,v_2),(v,v_n)\}$

$\widetilde{W}_n(W_n, R_1, R_2, \dots, R_{n-1}, R_n)$ is a fuzzy graph structure of wheel graph(Fig.2).

Here, $\{(v_2,v_3),(v_3,v_4),(v_4,v_5),\dots,(v_n,v_1),(v_1,v_2)\}$ form an R-independent set.

$\{(v,v_1),(v,v_2),(v,v_3),\dots,(v,v_{n-1}),(v,v_n)\}$ form another R-independent set.

There exist 2 R-independent sets. Hence, $\chi_{R_i}(\widetilde{W}_n) = 2$.

$\{(v_2,v_3),(v,v_1)\},\{(v_3,v_4),(v,v_2)\},\{(v_4,v_5),(v,v_3)\},\dots,\{(v_n,v_1),(v,v_{n-1})\},\{(v_1,v_2),(v,v_n)\}$ is a partition of the edge set into R −strongly independent sets. There exist n R-strongly independent sets. Hence, $\chi_{R_s}(\widetilde{W}_n) = n$.
For the membership function defined for the edges fuzzy graph structure of wheel graph, $\widetilde{W}_n$

$$\chi_{R_i}(\widetilde{W}_n) = 2, \chi_{R_s}(\widetilde{W}_n) = n$$

Theorem1.2: For the membership function defined for the edges fuzzy graph structure of Bistar graph $\tilde{B}_{m,n}$,
$\chi_{R_i}(\tilde{B}_{m,n}) = 2, \chi_{R_s}(\tilde{B}_{m,n}) = \begin{cases} n+1 \text{ if } m \le n \\ m+1 \text{ if } m > n' \end{cases}$

Proof:

Case:1

Consider $B_{m,n}(m<n)$ Consider $B_{m,n}$ (Figure3) The graph $B_{m,n}$ is a bistar obtained from copies of $K_{1,m}$ and $K_{1,n}$ by joining the centre vertices by an edge. It has $m+n+2$ vertices and $m+n+1$edges.

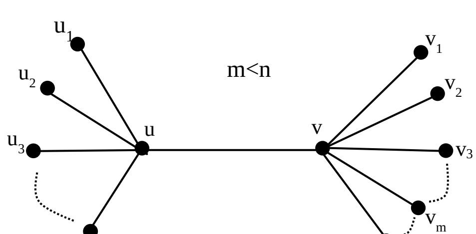

Figure 3

Let $\mu(v,v_1) = x_1$, $\mu(v,v_2) = x_2$, $\mu(v,v_3) = x_3, \dots$, $\mu(v,v_m) = x_m$, $\mu(v,v_{m+1}) = x_{m+1}, \dots, \mu(v,v_n) = x_n$
$\mu(u,v) = x_{n+1}$ $\mu(u,u_1) = x_1$, $\mu(u,u_2) = x_2$, $\mu(u,u_3) = x_3, \dots, \mu(u,u_m) = x_m$

The value of $x_1, x_2, x_3 \dots x_{n-1}, x_n$ lies in [0,1] and $\{x_1, x_2, x_3, \dots, x_{n-1}, x_n\}$ is strictly non -decreasing.

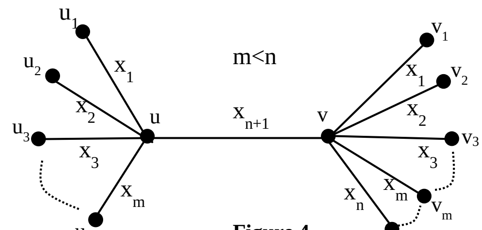

Figure 4

Group the edges with same membership functions

$R_1 = \{(v,v_1),(u,u_1)\}$,
$R_2 = \{(v,v_2),(u,u_2)\}$,
$R_3 = \{(v,v_3),(u,u_3)\}$,
$\vdots$
$R_m = \{(v,v_m),(u,u_m)\}$,
$R_{m+1} = \{(v,v_{m+1})\}$,
$R_{m+2} = \{(v,v_{m+2})\}$,
$\vdots$
$R_n = \{(v,v_n)\}$,
$R_{n+1} = \{(u,v)\}$

$(\tilde{B}_{m,n})$ $(B_{m,n}, R_1, R_2, R_3, \dots, R_m, R_{m+1} R_{m+2}, \dots, R_n, R_{n+1})$ is a fuzzy graph structure of bipartite graph(Fig.4).
Here,
$\{(v,v_1),(v,v_2),(v,v_3),\dots,(v,v_m),(v,v_{m+1})\},(v,v_{m+2})\},\dots,(v,v_n),(u,v)\}$ form an R-independent set.

Here, $\{(u,u_1)\}$, $(u,u_2)\}$, $(u,u_3)\},\dots,(u,u_m)\}$form another R-independent set.

There exist 2 R-independent sets. Hence, $\chi_{R_i}(\tilde{B}_{m,n})$= 2.

$\{(v,v_1),(u,u_1)\},\{(v,v_2),(u,u_2)\},\{(v,v_3),(u,u_3)\},\dots,\{(v,v_m),(u,u_m)\},\{(v,v_{m+1})\},\{(v,v_{m+2})\},\dots,$

$\{(v,v_n)\},\{(u,v)\}$ is a partition of the edge set into R-strongly independent sets. There exist $n+1$ R-strongly independent sets. Hence, $\chi_{R_s}(\tilde{B}_{m,n}) = n+1$.

For the membership function defined for the edges fuzzy graph structure of Bistar graph $\tilde{B}_{m,n}$,

$\chi_{R_i}(\tilde{B}_{m,n}) = 2, \chi_{R_s}(\tilde{B}_{m,n}) = n+1 \text{ if } m<n.$

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Case: 2

Consider $B_{m,n}$ $(m = n)$ Consider $B_{n,n}$ (Fig.5) The graph $B_{n,n}$ is a bistar obtained from copies of $K_{1,n}$ and $K_{1,n}$ by joining the centre vertices by an edge. It has $2n + 2$ vertices and $2n + 1$edges.

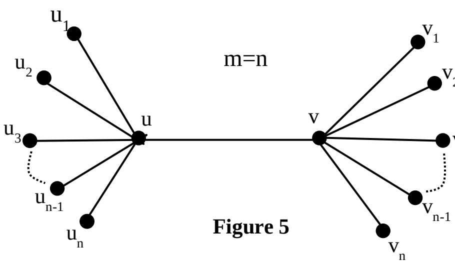

Figure 5

Let $\mu(v, v_1) = x_1$, $\mu(v, v_2) = x_2$, $\mu(v, v_3) = x_3,\ldots,$ $\mu(v, v_m) = x_m$, $\mu(v, v_{m+1}) = x_{m+1}, \ldots, \mu(v, v_n) = x_n$
$\mu(u, v) = x_{n+1}$ $\mu(u, u_1) = x_1$, $\mu(u, u_2) = x_2$,
$\mu(u, u_3) = x_3,\ldots, \mu(u, u_n) = x_n$

The value of $x_1, x_2, x_3 \ldots x_{n-1}, x_n$ lies in [0,1] and $\{x_1, x_2, x_3, \ldots, x_{n-1}, x_n\}$ is strictly non -decreasing.

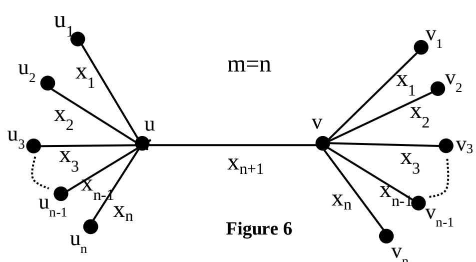

Figure 6

Group the edges with same membership functions

$R_1 = \{(v, v_1), (u, u_1)\}$,
$R_2 = \{(v, v_2), (u, u_2)\}$,
$R_3 = \{(v, v_3), (u, u_3)\}$,
⋮
$R_n = \{(v, v_n), (u, u_n)\}$,
$R_{n+1} = \{(u, v)\}$

$(\tilde{B}_{n,n})$ $(B_{n,n}$, $R_1, R_2, R_3, \ldots, R_{n-1}, R_n, R_{n+1})$ is a fuzzy graph structure of bipartite graph(Figure 6).

Here, $\{(v, v_1), (v, v_2), (v, v_3), \ldots, (v, v_{n-1}), (v, v_n)(u, v)\}$ form an R-independent set.

Here, $\{(u, u_1), (u, u_2)\}, (u, u_3), \ldots, (u, u_{n-1}), (u, u_n)\}$form another R-independent set.

There exist 2 R-independent sets. Hence, $\chi_{R_i}(\tilde{B}_{n,n}) = 2$
$\{(v, v_1), (u, u_1)\}, \{(v, v_2), (u, u_2)\}, \{(v, v_3), (u, u_3)\}, \ldots,$ $\{(v, v_n), (u, u_n)\}, \{(u, v\}$is a partition of the edge set into R-strongly independent sets. There exist $n + 1$R-strongly independent sets.

Hence, $\chi_{R_s}(\tilde{B}_{n,n}) = n + 1$.

For the membership function defined for the edges fuzzy graph structure of Bistar graph $\tilde{B}_{n,n}$

$\chi_{R_i}(\tilde{B}_{n,n}) = 2,\ \ \chi_{R_s}(\tilde{B}_{n,n}) = n + 1$

Case:3

Consider $B_{m,n}$ $(m > n)$ Consider $B_{m,n}$ (Figure7) The graph $B_{m,n}$ is a bistar obtained from copies of $K_{1,m}$ and $K_{1,n}$ by joining the centre vertices by an edge. It has $m + n + 2$ vertices and $m + n + 1$edges.

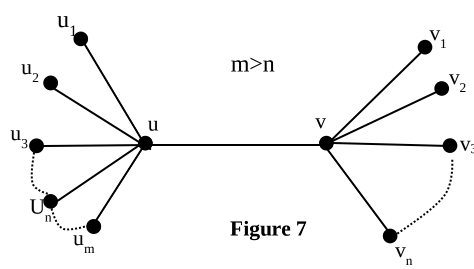

Figure 7

Let $\mu(v, v_1) = x_1$, $\mu(v, v_2) = x_2$, $\mu(v, v_3) = x_3,\ldots, \ldots, \mu(v, v_n) = x_n$, $\mu(u, u_1) = x_1$, $\mu(u, u_2) = x_2$, $\mu(u, u_3) = x_3,\ldots, \mu(u, u_n) = x_n$, $\mu(u, v) = x_{n+1}$

The value of $x_1, x_2, x_3 \ldots x_{n-1}, x_n$ lies in [0,1] and $\{x_1, x_2, x_3, \ldots, x_{n-1}, x_n\}$ is strictly non -decreasing.

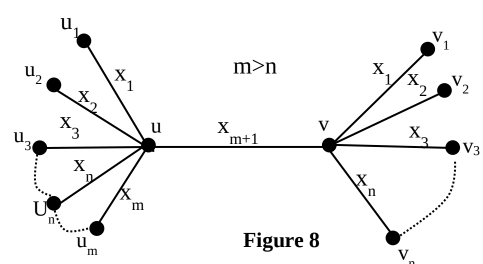

Figure 8

Group the edges with same membership functions

$R_1 = \{(v, v_1), (u, u_1)\}$,
$R_2 = \{(v, v_2), (u, u_2)\}$,
$R_3 = \{(v, v_3), (u, u_3)\}$,
⋮
$R_n = \{(v, v_n), (u, u_n)\}$,
$R_{n+1} = \{(u, u_{n+1})\}$,

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$R_{n+2} = \{(u, u_{n+2})\}$,
⋮
$R_m = \{(u, u_m)\}$,
$R_{m+1} = \{(u, v)\}$

$(\tilde{B}_{m,n})(B_{m,n}, R_1, R_2, \ldots, R_n, R_{n+1}, R_{n+2}, \ldots, R_m, R_{m+1})$ is a fuzzy graph structure of bipartite graph.

Here,$\{(v, v_1), (v, v_2), (v, v_3), \ldots, (v, v_n), (u, u_{n+1}), (u, u_{n+2})\}, \ldots, (u, u_m), (u, v)\}$ form an R-independent set.

Here, $\{(u, u_1)\}, (u, u_2)\}, (u, u_3)\}, \ldots, (u, u_n)\}$form another R-independent set.

There exist 2 R-independent sets. Hence, $\chi_{R_i}(\tilde{B}_{m,n}) = 2$. $\{(v, v_1), (u, u_1)\}, \{(v, v_2), (u, u_2)\}, \{(v, v_3), (u, u_3)\}, \ldots, \{(v, v_n), (u, u_n)\}, \{(u, u_{n+1})\}, \{(u, u_{n+2})\}, \ldots,$
$\{(u, u_m)\}, \{(u, v)\}$ is a partition of the edge set into R-strongly independent sets. There exist $m+1$ R-strongly independent sets. Hence, $\chi_{R_s}(\tilde{B}_{m,n}) = m+1$.

For the membership function defined for the edges fuzzy graph structure of

Bistar graph $\tilde{B}_{m,n}$,

$\chi_{R_i}(\tilde{B}_{m,n})$=2, $\chi_{R_s}(\tilde{B}_{m,n})$= $m+1 \; if \; m > n$.

For the membership function defined for the edges fuzzy graph structure of Bistar graph $(\tilde{B}_{m,n})\chi_{R_i}(\tilde{B}_{m,n})$=2,

$$\chi_{R_s}(\tilde{B}_{m,n}) = \begin{cases} n+1 \; if \; m \le n \\ m+1 \; if \; m > n \end{cases}.$$

III. CONCLUSION

R-independent edge colouring number χ_{R_i} of a fuzzy graph structure and the strong R-edge colouring number χ_{R_s} of a fuzzy graph structure are computed for fuzzy wheel graph structure and fuzzy bistar graph structure.

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